Transceivers based on the Ideal of Network Coding

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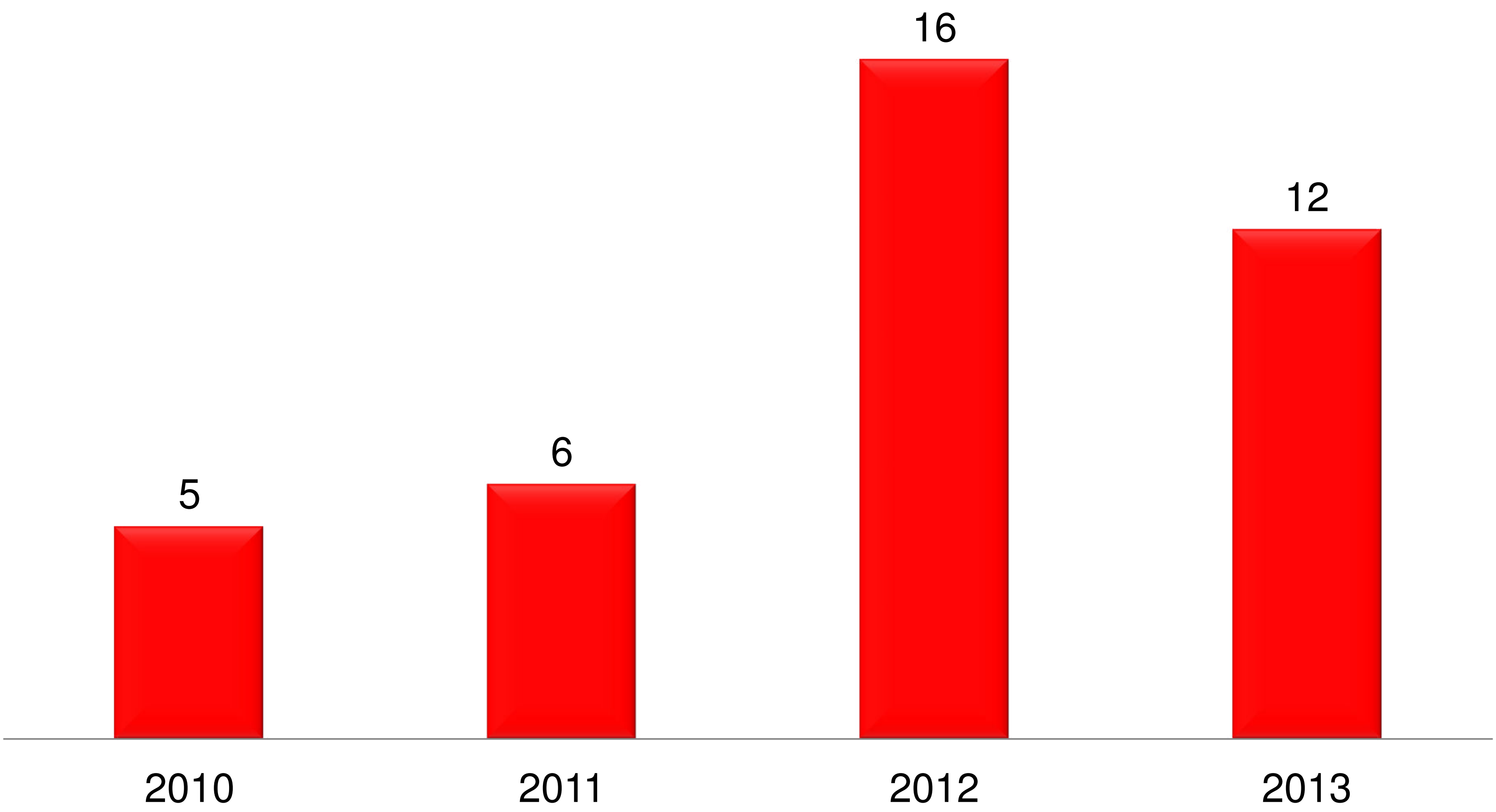

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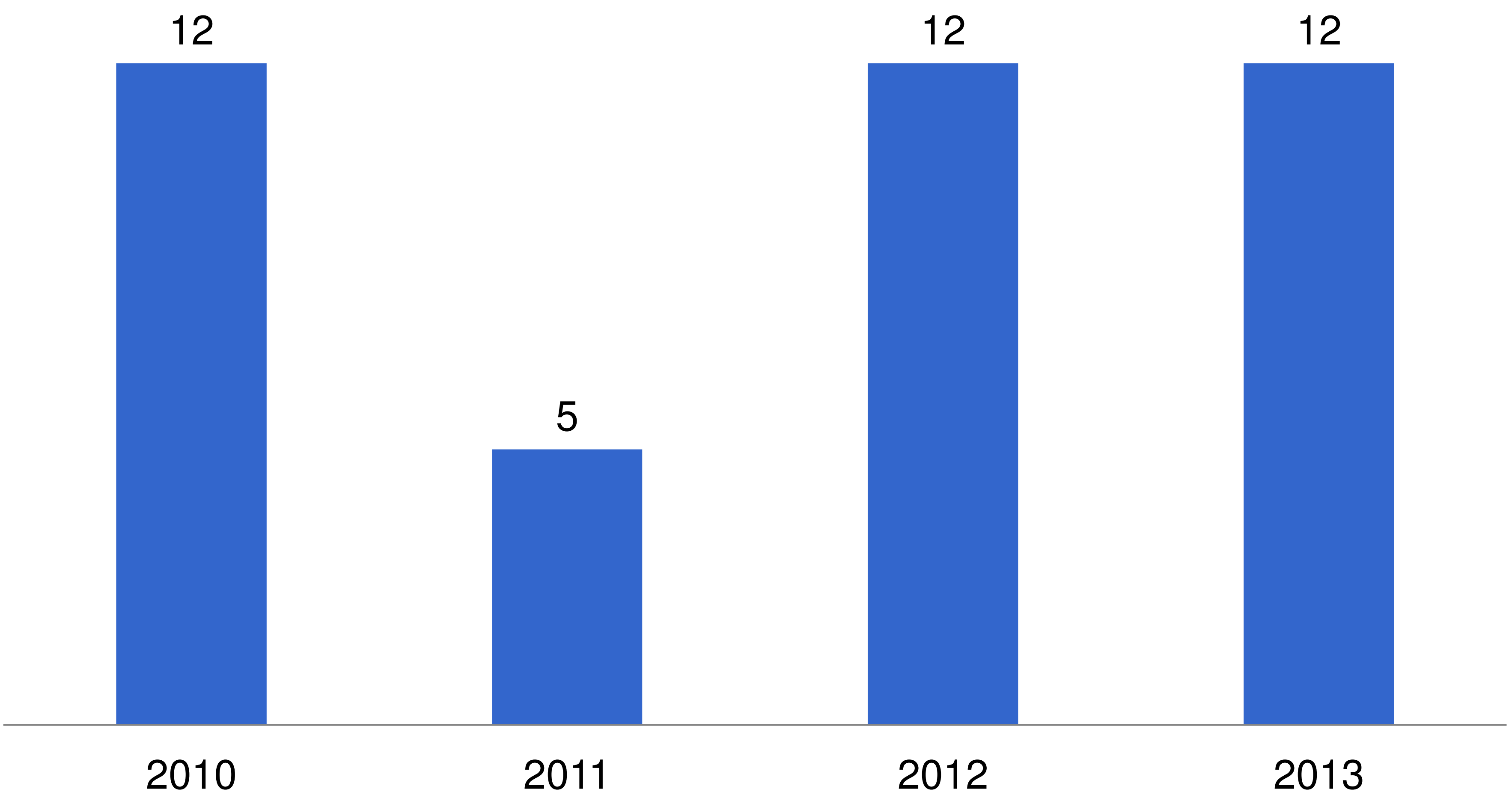

Recent Grants

- NSFC Projects
  - 多源多宿无线多中继网络的分集-复用分析与网络纠错码的设计
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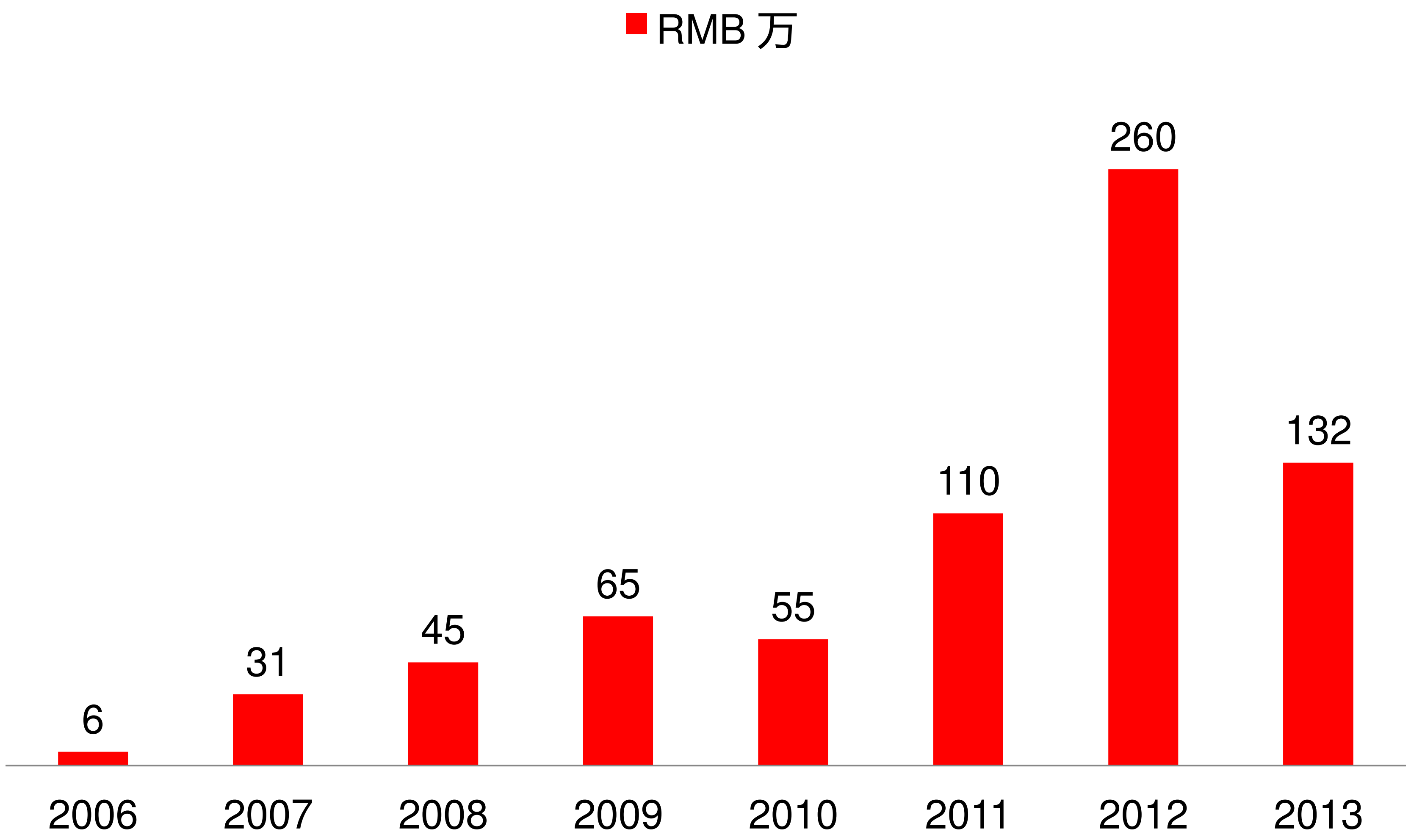
Funding Amounts per Year



Research Area

- Coding
  - Network Coding
  - Channel Coding
- Relay Networks
- MIMO-OFDM Systems
- Cognitive Radio
- Green Communications
- Heterogeneous Networks

Research Background

- Network Coding (NC)
  Originally designed for wired networks, network coding is a generalized routing approach that allows intermediate nodes to send out functions of their received packets, by which the multicast capacity given by max-flow min-cut theorem can be achieved. [Ahlswede, Cai, Li, Yeung, 2000]
- Linear Network Coding
  For multicasting, intermediate nodes can simply send out a linear combination of their received packets to achieve the capacity. [Li, Ahlswede, Cai, 2003], [Koetter, Medard, 2003]
- Physical Layer Network Coding (PLNC)
  In order to address the broadcast nature of wireless transmission, PLNC was proposed to embrace interference in wireless networks in which intermediate nodes attempt to decode the modulo-two sum (XOR) of the transmitted messages. [Zhang, Liew, Lam, 2006]
- Analog Network Coding (ANC)
  A relay will simply amplify-and-forward mixed signals. [Katti, Gollakota, Katabi, 2007]
- Compute-and-Forward (CPF)
  CPF is a promising new approach to PLNC for general wireless networks, beneficial from both network coding and lattice codes. The main idea is that a relay will decode a linear function of transmitted messages according to the observed channel coefficients. [Nazer, Gastpar, 2011]

System Model

• System Model

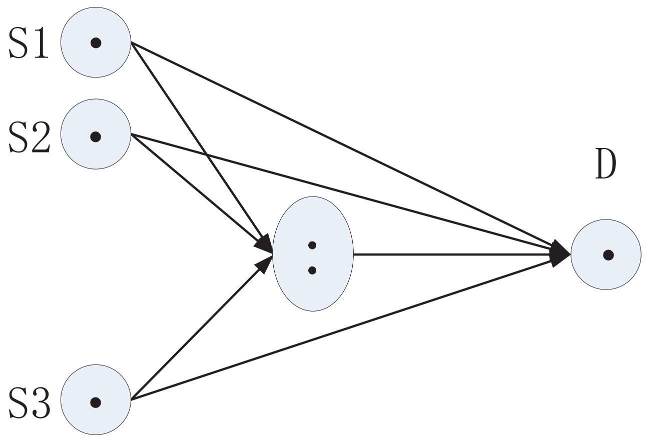


Figure 1: Three sources with direct links

- Three sources are communicating to one destination through one relay without direct links.
- We assume the sources and the destination are equipped with single antenna, while the relay is equipped with two antennas.
- The information transmission is performed in two phases with three time slots in total.
- In the first phase three source nodes transmit simultaneously to relay $\mathcal{R}$ in one time slot; while in the second phase relay $\mathcal{R}$ transmits to destination $\mathcal{D}$ in the remaining two time slots.

Different Schemes

- **Scheme 1: Direct Transmission (DT)**

(i) In this scheme, we assume the relay will keep silent and the sources will communicate to the destination one by one.

(ii) Let f_i be the direct link channel coefficient between source $\mathcal{S}_i$ to destination $\mathcal{D}$; x_i be the transmit signal from node $\mathcal{S}_i$ which satisfies the power constraint $E\{|x_i|^2\} \leq P_x$.

(iii) The received signals at destination $\mathcal{D}$ during three time slots are

$$y_{D1} = f_1 x_1 + n_{D1}, \quad (1)$$

$$y_{D2} = f_2 x_2 + n_{D2}, \quad (2)$$

$$y_{D3} = f_3 x_3 + n_{D3}, \quad (3)$$

which can be combined to

$$\mathbf{y}_{DT} = \underbrace{\begin{bmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{bmatrix}}_{\mathbf{A}_{DT}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} n_{D1} \\ n_{D2} \\ n_{D3} \end{bmatrix}}_{\mathbf{z}_{DT}}. \quad (4)$$

(iv) The decoding procedure for DT scheme will simply be

$$\hat{\mathbf{x}}_{DT} = \arg\min_{\mathbf{x}\in\Omega_{\mathbf{x}}} ||\mathbf{y}_{DT} - \mathbf{A}_{DT}\mathbf{x}||^2. \qquad (5)$$

$\mathbf{x}$ is the transmit data vector of three sources $\mathbf{x} \triangleq [x_1, x_2, x_3]^T$ and $\mathbf{x} \in \Omega_{\mathbf{x}}$, where $\Omega_{\mathbf{x}}$ is the data vector alphabet set.

(v) The sum rate at destination $\mathcal{D}$ for DT scheme will be

$$R_{DT} = \frac{1}{3} \log\det\left(\mathbf{I}_3 + P_x \mathbf{A}_{DT}\mathbf{A}_{DT}^H\right). \qquad (6)$$

The one-third factor above is the natural consequence of time sharing.

- **Scheme 2: Analog Network Coding (ANC)**

(i) Regarding this scheme, in the first phase all source nodes transmit simultaneously to relay $\mathcal{R}$ and destination $\mathcal{D}$ in one time slot; while the second phase is the transmission from relay $\mathcal{R}$ to destination $\mathcal{D}$ during the remaining two time slots.

(ii) At the end of first phase, the received signal at destination $\mathcal{D}$ is

$$y_D^{[1]} = [f_1, f_2, f_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + n_D^{[1]} = \mathbf{f}^T \mathbf{x} + n_D^{[1]}, \tag{7}$$

where superscript $\{\cdot\}^{[1]}$ denotes the first phase; the direct link channel vector $\mathbf{f} \triangleq [f_1, f_2, f_3]^T$.

(iii) The received signal at relay $\mathcal{R}$ at the end of first phase is

$$\mathbf{y}_R = \begin{bmatrix} y_{R1} \\ y_{R2} \end{bmatrix} = \underbrace{\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \mathbf{n}_R = \mathbf{H}\mathbf{x} + \mathbf{n}_R. \tag{8}$$

(iv) We denote the channel vector of all sources to relay antenna r as

$$\mathbf{h}_r = [h_{r1}, h_{r2}, h_{r3}]^T \in \mathbb{C}^3. \tag{9}$$

In the second phase, first, relay $\mathcal{R}$ constructs the following signal vector $\mathbf{t}$,

$$\mathbf{t} = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \beta_1 y_{R1} \\ \beta_2 y_{R2} \end{bmatrix} = \underbrace{\begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix}}_{\mathbf{B}} (\mathbf{Hx} + \mathbf{n}_R). \tag{10}$$

where β_r, $r = 1, 2$ is the scaling factor given by

$$\beta_r = \sqrt{\frac{P_R}{E\{|y_{Rr}|^2\}}} = \sqrt{\frac{P_R}{P_x||\mathbf{h}_r||^2 + 1}}. \tag{11}$$

(v) Then, relay $\mathcal{R}$ will transmit t_1 and t_2 in two time slots,

$$\left[y_D^{[2]}(1), y_D^{[2]}(2)\right] = [g_1, g_2] \begin{bmatrix} t_1 & 0 \\ 0 & t_2 \end{bmatrix} + [n_D^{[2]}(1), n_D^{[2]}(2)], \tag{12}$$

where superscript $\{\cdot\}^{[2]}$ denotes the second phase; g_r, $r = 1, 2$, is the channel coefficient between relay antenna r and destination $\mathcal{D}$.

Equivalently, equation (12) can be written as

$$\mathbf{y}_D^{[2]} = \begin{bmatrix} y_D^{[2]}(1) \\ y_D^{[2]}(2) \end{bmatrix} = \begin{bmatrix} g_1 & 0 \\ 0 & g_2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} + \mathbf{n}_D^{[2]} \tag{13}$$

$$= \mathbf{G}_0\mathbf{t} + \mathbf{n}_D^{[2]}, \tag{14}$$

$$= \mathbf{G}_0\mathbf{BHx} + \mathbf{G}_0\mathbf{Bn}_R + \mathbf{n}_D^{[2]}. \tag{15}$$

(vi) After one transmission realization, we can combine received signals at destination $\mathcal{D}$ during two phases (three time slots), based on which to decode the data vector $\mathbf{x}$, as follows,

$$\mathbf{y}_{ANC} = \begin{bmatrix} y_D^{[1]} \\ \\ \mathbf{y}_D^{[2]} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{f}^T \\ \\ \mathbf{G}_0\mathbf{BH} \end{bmatrix}}_{\mathbf{A}_{ANC}} \mathbf{x} + \underbrace{\begin{bmatrix} n_D^{[1]} \\ \mathbf{G}_0\mathbf{Bn}_R + \mathbf{n}_D^{[2]} \end{bmatrix}}_{\mathbf{z}_{ANC}}. \tag{16}$$

(vii) Hence the decoding procedure for ANC scheme will be

$$\hat{\mathbf{x}}_{ANC} = \arg\min_{\mathbf{x}\in\Omega_{\mathbf{x}}} ||\mathbf{y}_{ANC} - \mathbf{A}_{ANC}\mathbf{x}||^2. \tag{17}$$

(viii) Let $\mathbf{K}_{ANC}$ be the covariance matrix of effective noise vector $\mathbf{z}_{ANC}$ at destination, i.e.,

$$\begin{aligned} \mathbf{K}_{ANC} &\triangleq \mathbb{E}\left\{\mathbf{z}_{ANC}\,\mathbf{z}_{ANC}^{H}\right\} \\ &= \begin{bmatrix} 1 & \mathbf{0}_2^T \\ \mathbf{0}_2 & \mathbf{G}_0\mathbf{B}\mathbf{B}^H\mathbf{G}_0^H + \mathbf{I}_2 \end{bmatrix} \end{aligned} \tag{18}$$

where $\mathbb{E}\{\cdot\}$ is the expectation operation; $\mathbf{0}_2 = [0, 0]^T$ is the all-zero column vector in two dimension.

The sum rate at destination $\mathcal{D}$ for ANC scheme will be

$$R_{ANC} = \frac{1}{3}\log\det\left(\mathbf{I}_3 + P_x\,\mathbf{A}_{ANC}\,\mathbf{A}_{ANC}^{H}\,\mathbf{K}_{ANC}^{-1}\right). \tag{19}$$

- **Scheme 3: Space-Time Analog Network Coding with Alamouti (STANC-Alamouti)**

(i) In this scheme, the first transmission phase will be the same as in ANC scheme.

(ii) After constructing the signal vector $\mathbf{t} = [t_1, t_2]^T$ as equation (10), relay $\mathcal{R}$ will combine analog network coding with Alamouti scheme. Relay $\mathcal{R}$ will transmit $[t_1, t_2]^T$ in the second time slot and $[-t_2^*, t_1^*]^T$ in the third time slot.

$$\left[y_D^{[2]}(1), y_D^{[2]}(2)\right] = [g_1, g_2] \begin{bmatrix} t_1 & -t_2^* \\ t_2 & t_1^* \end{bmatrix} + [n_D^{[2]}(1), n_D^{[2]}(2)]. \tag{20}$$

(iii) Destination $\mathcal{D}$ arranges the received signals into a vector $\mathbf{y}_D^{[2]} = \left[y_D^{[2]}(1), -y_D^{[2]}(2)^*\right]^T$, which can be rewritten as

$$\mathbf{y}_D^{[2]} = \begin{bmatrix} y_D^{[2]}(1) \\ -y_D^{[2]}(2)^* \end{bmatrix} = \begin{bmatrix} g_1 & g_2 \\ -g_2^* & g_1^* \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} + \mathbf{n}_D^{[2]} \tag{21}$$

$$= \mathbf{G}_1 \mathbf{t} + \mathbf{n}_D^{[2]} \tag{22}$$

$$= \mathbf{G}_1 \mathbf{B} \mathbf{H} \mathbf{x} + \mathbf{G}_1 \mathbf{B} \mathbf{n}_R + \mathbf{n}_D^{[2]}. \tag{23}$$

(iv) Finally, after one transmission realization, we combine received signals at destination $\mathcal{D}$ during two phases (three time slots) as

$$\mathbf{y}_{\mathsf{STANC}} = \begin{bmatrix} y_D^{[1]} \\ \\ \mathbf{y}_D^{[2]} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{f}^T \\ \\ \mathbf{G}_1\mathbf{B}\mathbf{H} \end{bmatrix}}_{\mathbf{A}_{\mathsf{STANC}}} \mathbf{x} + \underbrace{\begin{bmatrix} n_D^{[1]} \\ \mathbf{G}_1\mathbf{B}\mathbf{n}_R + \mathbf{n}_D^{[2]} \end{bmatrix}}_{\mathbf{z}_{\mathsf{STANC}}}. \quad (24)$$

(v) The decoding procedure can be expressed as

$$\hat{\mathbf{x}}_{\mathsf{STANC}} = \arg\min_{\mathbf{x}\in\Omega_{\mathbf{x}}} ||\mathbf{y}_{\mathsf{STANC}} - \mathbf{A}_{\mathsf{STANC}}\mathbf{x}||^2. \quad (25)$$

(vi) Let $\mathbf{K}_{\mathsf{STANC}}$ be the covariance matrix of effective noise vector $\mathbf{z}_{\mathsf{STANC}}$ at destination, i.e.,

$$\mathbf{K}_{\mathsf{STANC}} \triangleq \mathbb{E}\left\{\mathbf{z}_{\mathsf{STANC}}\,\mathbf{z}_{\mathsf{STANC}}^H\right\} = \begin{bmatrix} 1 & \mathbf{0}_2^T \\ \\ \mathbf{0}_2 & \mathbf{G}_1\mathbf{B}\mathbf{B}^H\mathbf{G}_1^H + \mathbf{I}_2 \end{bmatrix}. \quad (26)$$

The sum rate at destination $\mathcal{D}$ for STANC scheme will be

$$R_{\mathsf{STANC}} = \frac{1}{3}\log\det\left(\mathbf{I}_3 + P_x\,\mathbf{A}_{\mathsf{STANC}}\,\mathbf{A}_{\mathsf{STANC}}^H\,\mathbf{K}_{\mathsf{STANC}}^{-1}\right). \quad (27)$$

Table 1: Different Schemes

	Time Slot 1	Time Slot 2	Time Slot 3
DT	$S_1 : x_1$	$S_2 : x_2$	$S_3 : x_3$
ANC	$S_1 : x_1$ $S_2 : x_2$ $S_3 : x_3$	$R : \begin{bmatrix} t_1 \\ 0 \end{bmatrix}$	$R : \begin{bmatrix} 0 \\ t_2 \end{bmatrix}$
STANC-Alamouti	$S_1 : x_1$ $S_2 : x_2$ $S_3 : x_3$	$R : \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$	$R : \begin{bmatrix} -t_2^* \\ t_1^* \end{bmatrix}$

Simulation Studies

- **Simulation Comparison for Different Schemes**

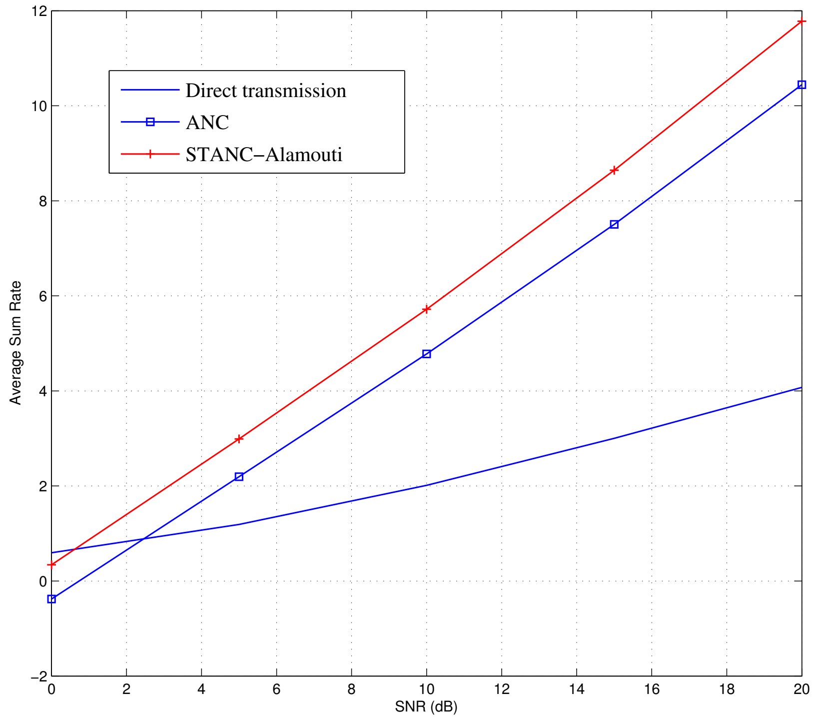


Figure 2: Sum Rate Comparison for Different Schemes

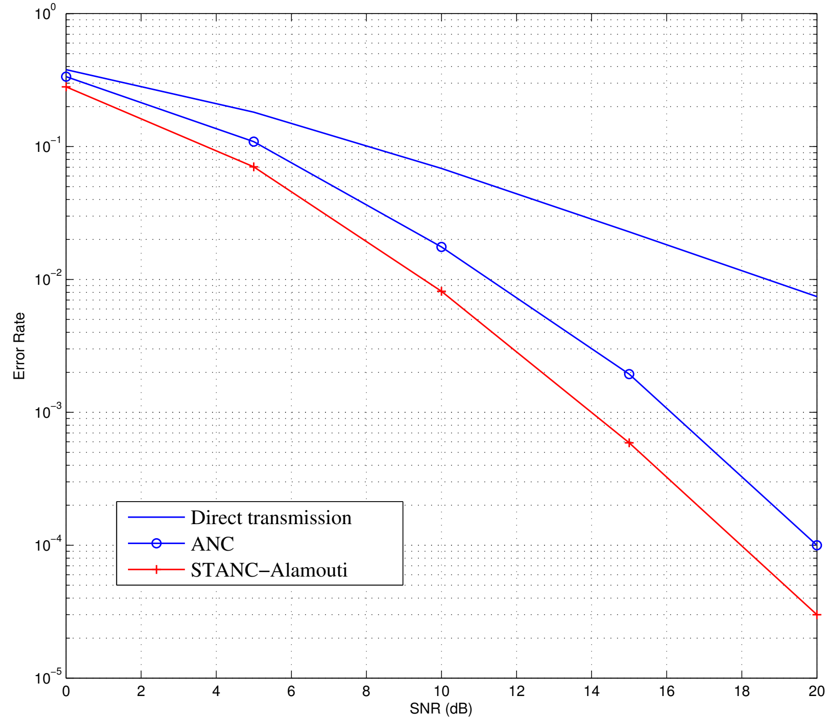


Figure 3: BER Comparison for Different Schemes

Conclusions

- We investigate space-time analog network coding (STANC) in multiple-access relay channels (MARC) system model, where three sources communicate to a common destination through a two-antenna relay with direct links.

- We discuss several possible transmission schemes under three time slots constraint: *(i)* direct transmission (DT); *(ii)* analog network coding (ANC); *(iii)* space-time analog network coding with alamouti scheme (STANC-Alamouti).

- Simulation studies show that STANC with alamouti scheme outperform other schemes regarding sum rate and bit error rate performance at the destination.

System Model and Notations

- We consider the classic MIMO channel with L transmit antennas and N receive antennas. Each transmit antenna delivers an independent data stream which encoded separately to form the transmitted codewords.

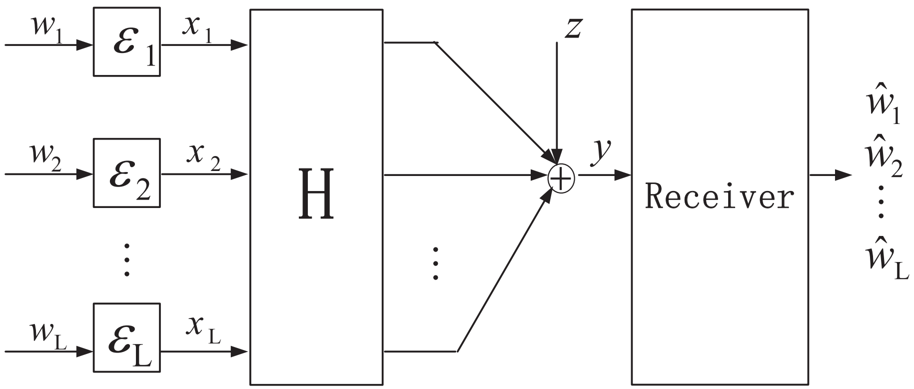


Figure 1: System Diagram

- Each antenna has a length-k information vector $\mathbf{w}_m \in \mathbb{F}_p^k$,

$$\mathbf{w}_m = [w_m(1), w_m(2), \cdots, w_m(k)]. \quad (1)$$

The encoder, $\mathcal{E}_m : \mathbb{F}_p^k \to \mathbb{R}^n$, maps the length-$k$ message $\mathbf{w}_m$ into a length-n lattice codeword $\mathbf{x}_m \in \mathbb{R}^n$, which satisfies the power constraint of $\frac{1}{n}||\mathbf{x}_m||^2 \leq P$,

$$\mathbf{x}_m = [x_m(1), x_m(2), \cdots, x_m(n)]. \quad (2)$$

System Model and Notations (cntd.)

- In the ith transmission realization, the received vector is,

$$\mathbf{y}^{[i]} = \sum_{m=1}^{L} \mathbf{h}_m x_m(i) + \mathbf{z}^{[i]} = \mathbf{H}\mathbf{x}^{[i]} + \mathbf{z}^{[i]}, \tag{3}$$

  where

$$\mathbf{x}^{[i]} = [x_1(i), x_2(i), \cdots, x_L(i)]^T; \tag{4}$$

  $\mathbf{h}_m \in \mathbb{R}^N$ is real valued fading channel vector from antenna m to the receiver; the equivalent channel matrix $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_L]$.

- In a linear receiver architecture, the receiver will project $\mathbf{y}^{[i]}$ with some matrix $\mathbf{B} \in \mathbb{R}^{L\times N}$ to get the effective received vector for further decoding,

$$\tilde{\mathbf{y}}^{[i]} = \mathbf{B}\mathbf{y}^{[i]} = \mathbf{B}\mathbf{H}\mathbf{x}^{[i]} + \mathbf{B}\mathbf{z}^{[i]} = \mathbf{A}\mathbf{x}^{[i]} + \tilde{\mathbf{z}}^{[i]}. \tag{5}$$

- The standard linear detection methods include ZF receiver and MMSE receiver,

$$\begin{aligned} \mathbf{B}_{ZF} &= (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T & \mathbf{A}_{ZF} &= \mathbf{I}_L \\ \mathbf{B}_{MMSE} &= (\mathbf{H}^T\mathbf{H} + \tfrac{1}{P}\mathbf{I}_L)^{-1}\mathbf{H}^T & \mathbf{A}_{MMSE} &= \mathbf{B}_{MMSE}\mathbf{H}. \end{aligned} \tag{6}$$

System Model and Notations (cntd.)

- We recall the important algebraic structure of lattice codes, that the integer combination of lattice codewords is still a codeword. Integer forcing (IF) receiver [a] tries to design an equalization matrix $\mathbf{B}_{IF} \in \mathbb{R}^{L\times N}$, such that after projection, the resulting IF matrix $\mathbf{A}_{IF}$ satisfies that $\mathbf{A}_{IF} \in \mathbb{Z}^{L\times L}$ and the achievable rate is maximized.
- We summarize the existing results in [1]-[2] regarding IF receiver in the following theorem.

Theorem 1: *Let* $\mathbf{A}_{IF} = [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_L]^T$ *and* $\mathbf{B}_{IF} = [\mathbf{b}_1, \mathbf{b}_2, \cdots, \mathbf{b}_L]^T$*. For each pair of* $(\mathbf{a}_m, \mathbf{b}_m)$*, the following computation rate is achievable,*

$$\mathscr{R}_m = \frac{1}{2}\log\left(\frac{P}{||\mathbf{b}_m||^2 + P||\mathbf{H}^T\mathbf{b}_m - \mathbf{a}_m||^2}\right). \tag{7}$$

For a fixed IF coefficient matrix $\mathbf{A}_{IF}$*, the computation rate is maximized by choosing*

$$\mathbf{b}_m^T = \mathbf{a}_m^T\mathbf{H}^T\left(\mathbf{H}\mathbf{H}^T + \frac{1}{P}\mathbf{I}_L\right)^{-1}. \tag{8}$$

□

[a] [1] J. Zhan, B. Nazer, U. Erez and M. Gastpar, "Integer-forcing linear receivers", in *Proc. IEEE Inter. Symp. Info. Theory*, pp. 1022-1026, Austin, Texas, June 2010.

[2] J. Zhan, B. Nazer, U. Erez and M. Gastpar, "Integer-forcing linear receivers: a new low-complexity MIMO architecture", in *Proc. IEEE Veh. Tech. Conf.*, Ottawa, Canada, Sept. 2010.

System Model and Notations (cntd.)

- According to Theorem 1, we plug in the optimal $\mathbf{b}_m$ of (8) into the computation rate $\mathscr{R}_m$ of (7),

$$\mathscr{R}_m = \frac{1}{2}\log\left(\frac{1}{\mathbf{a}_m^T\,\mathbf{Q}\,\mathbf{a}_m}\right), \tag{9}$$

where

$$\mathbf{Q} \triangleq \mathbf{I}_L - \mathbf{H}^T\left(\mathbf{H}\mathbf{H}^T + \frac{1}{P}\mathbf{I}_L\right)^{-1}\mathbf{H}. \tag{10}$$

Then, the total achievable rate of the IF receiver is

$$\mathscr{R}_{total} \triangleq \max_{|\mathbf{A}|\neq 0} L \min_m \mathscr{R}_m = \max_{|\mathbf{A}|\neq 0}\min_m L\log\left(\frac{1}{\mathbf{a}_m^T\,\mathbf{Q}\,\mathbf{a}_m}\right), \tag{11}$$

- Hence, the design criteria for optimal IF coefficient matrix $\mathbf{A}_{IF}$ is

$$\mathbf{A}_{IF} = arg\max_{|\mathbf{A}|\neq 0}\min_m \frac{L}{2}\log\left(\frac{1}{\mathbf{a}_m^T\,\mathbf{Q}\,\mathbf{a}_m}\right) = arg\min_{|\mathbf{A}|\neq 0}\max_m \mathbf{a}_m^T\,\mathbf{Q}\,\mathbf{a}_m. \tag{12}$$

It means that we need to find integer vectors $\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_L$ to construct a full rank matrix $\mathbf{A}_{IF}$, such that the maximum value of $\mathbf{a}_m^T\,\mathbf{Q}\,\mathbf{a}_m$ is minimized.

Proposed Algorithms

- To approach the optimization problem of (12), first we need to generate some feasible searching set

$$\Omega \subset \mathbb{Z}^L, \tag{13}$$

to search $\mathbf{a}_m \in \Omega$, instead of the whole searching space $\mathbf{a}_m \in \mathbb{Z}^L$. Then, we will find L linearly independent vectors within this searching set Ω to construct the optimal IF coefficient matrix $\mathbf{A}_{IF}$.

- Accordingly, we propose the following strategy with two steps.
  - In the first step, we generate the searching set Ω based on Fincke-Pohst (FP) method [a], such that the integer vectors $\mathbf{t} \in \mathbb{Z}^L$ with top $|\Omega|$ minimum $\mathbf{t}^T \mathbf{Q} \mathbf{t}$ values are within.
  - In the second step, we pick up $\mathbf{a}_1$, $\mathbf{a}_2$, $\cdots$, $\mathbf{a}_L \in \Omega$, to construct the full rank IF coefficient matrix

$$\mathbf{A}_{IF} = [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_L]^T, \tag{14}$$

while in the meantime, the maximum value of $\mathbf{a}_m^T \mathbf{Q} \mathbf{a}_m$ is minimized. Then, equivalently, this optimal $\mathbf{A}_{IF}$ will maximize the total achievable rate.

[a] [3] U. Fincke and M. Pohst, "Improved methods for calculating vectors of short length in a lattice, including a complexity analysis," *Math. Comput.*, vol. 44, pp. 463-471, Apr. 1985.

Proposed Algorithms (cntd.)

A. FP Based Candidate Set Searching Algorithm

- We attempt to find the candidate set Ω such that integer vectors with top $|\Omega|$ minimum $\mathbf{t}^T\,\mathbf{Q}\,\mathbf{t}$ values are within. The procedure of enumerating all vectors $\mathbf{t} \in \mathbb{Z}^L$ $(\mathbf{t} \neq \mathbf{0})$ in Ω, such that

$$\mathbf{t}^T\mathbf{Q}\,\mathbf{t} \leq C, \tag{15}$$

for a given positive constant C is based on FP method.

- Regarding the positive constant C, we set it based on the binary vector obtained by applying the direct sign operator of the real minimum-eigenvalue eigenvector of $\mathbf{Q}$, denoted as $\mathbf{t}_{quant}$, such that

$$C = \mathbf{t}_{quant}^T\mathbf{Q}\,\mathbf{t}_{quant}. \tag{16}$$

By setting the searching sphere radius this way, it is big enough to have several searching vectors falls inside, while in the meantime small enough to have not too many searching vectors within.

Proposed Algorithms (cntd.)

Algorithm 1 FP Based Candidate Set Searching Algorithm

Input: Matrix $\mathbf{Q}$.

Output: The searching candidate set Ω.

Step 1: Calculate the binary quantized vector obtained by applying the direct sign operator of the real minimum-eigenvalue eigenvector of $\mathbf{Q}$, denoted as $\mathbf{t}_{quant}$, and set C as

$$C = \mathbf{t}_{quant}^T \mathbf{Q}\, \mathbf{t}_{quant}. \tag{17}$$

Step 2: Performing Cholesky factorization of matrix $\mathbf{Q}$ yields $\mathbf{Q} = \mathbf{U}^T\mathbf{U}$, where $\mathbf{U}$ is an upper triangular matrix. Let u_{ij}, $i, j = 1, 2, \cdots, L$ denote the entries of matrix $\mathbf{U}$. Set

$$g_{ii} = u_{ii}^2, \qquad g_{ij} = u_{ij}/u_{ii}, \tag{18}$$

for $i = 1, 2, \cdots, L$, $j = i + 1, \cdots, L$.

Step 3: Construct search set

$$\Omega = \left\{\mathbf{t} : \ \mathbf{t}^T\mathbf{Q}\,, \mathbf{t} \leq C, \mathbf{t} \neq \mathbf{0}, \ \mathbf{t} \in \mathbb{Z}^L\right\}, \tag{19}$$

according to the following FP procedure.

(i) Start from $\Delta_L = 0$, $C_L = C$, $k = L$ and $\Omega = \emptyset$.

(ii) Set the upper bound UB_k and the lower bound LB_k as follows

$$UB_k = \left\lfloor \sqrt{\frac{C_k}{g_{kk}}} - \Delta_k \right\rfloor, \qquad LB_k = \left\lceil -\sqrt{\frac{C_k}{g_{kk}}} - \Delta_k \right\rceil \tag{20}$$

and $t_k = LB_k - 1$.

(iii) Set $t_k = t_k + 1$. For $t_k \leq UB_k$, go to (v); else go to (iv).

(iv) If $k = L$, terminate and output Ω; else set $k = k + 1$ and go to (iii).

(v) For $k = 1$, go to (vi); else set $k = k - 1$, and

$$\Delta_k = \sum_{j=k+1}^{L} g_{kj} t_j, \tag{21}$$

$$C_k = C_{k+1} - g_{k+1,k+1} \left(\Delta_{k+1} + t_{k+1}\right)^2 \tag{22}$$

then go to (ii).

(vi) If $\mathbf{t} = \mathbf{0}$ terminate, else we get a candidate vector $\mathbf{t} \neq \mathbf{0}$ that satisfies all the bounds requirements and put it inside Ω, i.e. $\Omega = \{\Omega, \mathbf{t}\}$. Go to (iii).

Proposed Algorithms (cntd.)

B. Constructing IF Coefficient Matrix $\mathbf{A}_{IF}$

- According to our proposed Algorithm 1, we get the feasible searching set Ω. Define a function $f(\mathbf{t}) \triangleq \mathbf{t}^T\mathbf{Q}\mathbf{t}$. We sort the vectors in the searching set such that

$$\Omega = \{\mathbf{t}^{[1]}, \mathbf{t}^{[2]}, \cdots, \mathbf{t}^{[|\Omega|]} : f(\mathbf{t}^{[1]}) \leq f(\mathbf{t}^{[2]}) \leq \cdots \leq f(\mathbf{t}^{[|\Omega|]})\}. \tag{23}$$

- Choose L linear independent vectors within this sorted set by

$$\mathbf{a}_1 = \mathbf{t}^{[i_1]}, \quad \mathbf{a}_2 = \mathbf{t}^{[i_2]}, \quad \cdots, \mathbf{a}_L = \mathbf{t}^{[i_L]}, \tag{24}$$

for some $i_1 < i_2 < \cdots < i_L$. Then, the optimization of (12) becomes

$$\mathbf{A}_{IF} = arg \min_{|\mathbf{A}|\neq 0} \max_{m} \mathbf{a}_m^T \mathbf{Q}\, \mathbf{a}_m = arg \min_{|\mathbf{A}|\neq 0} \mathbf{a}_L^T \mathbf{Q}\, \mathbf{a}_L. \tag{25}$$

Hence, we attempt to find the last coefficient vector $\mathbf{a}_L$, such that $\mathbf{a}_L^T \mathbf{Q}\, \mathbf{a}_L$ is minimized, if $\mathbf{a}_1$, $\mathbf{a}_2$, $\cdots$, $\mathbf{a}_{L-1}$ are chosen from the sorted set Ω with vectors in front of $\mathbf{a}_L$.

Proposed Algorithms (cntd.)

Algorithm 2 IF Coefficient Matrix Constructing Algorithm

Input: Searching set Ω.

Output: The IF coefficient matrix $\mathbf{A}_{IF}$ with full rank that gives the maximum total achievable rate.

Step 1: Define a function $f(\mathbf{t}) \triangleq \mathbf{t}^T\mathbf{Q}\mathbf{t}$ and sort the vectors in the searching set such that

$$\Omega = \{\mathbf{t}^{[1]}, \mathbf{t}^{[2]}, \cdots, \mathbf{t}^{[|\Omega|]} : f(\mathbf{t}^{[1]}) \leq f(\mathbf{t}^{[2]}) \leq \cdots \leq f(\mathbf{t}^{[|\Omega|]})\}. \quad (26)$$

Initiate $i_L = L$.

Step 2: If $i_L > |\Omega|$, go to Step 4. Else, let $\mathbf{a}_L = \mathbf{t}^{[i_L]}$. Construct the cut set

$$\Omega_{cut} = \{\mathbf{t}^{[1]}, \mathbf{t}^{[2]}, \cdots, \mathbf{t}^{[i_L-1]}\}. \quad (27)$$

Then, search through $\binom{i_L-1}{L-1}$ possibilities, to see whether we can find $\mathbf{a}_1$, $\mathbf{a}_2$, $\cdots$, $\mathbf{a}_{L-1} \in \Omega_{cut}$ such that the constructed $\mathbf{A}_{IF}$ is of full rank.

Step 3: Once we find one full rank matrix $\mathbf{A}_{IF}$, terminate and output this $\mathbf{A}_{IF}$. Else, $i_L = i_L + 1$ and go to Step 2.

Step 4: If we cannot construct full rank matrix $\mathbf{A}_{IF}$ within searching set Ω, we expand the C value setting in (17) as $C = 2C$ and re-generate the searching set Ω by our proposed FP Based Candidate Set Searching Algorithm. Then, go to Step 1.

Experimental Studies

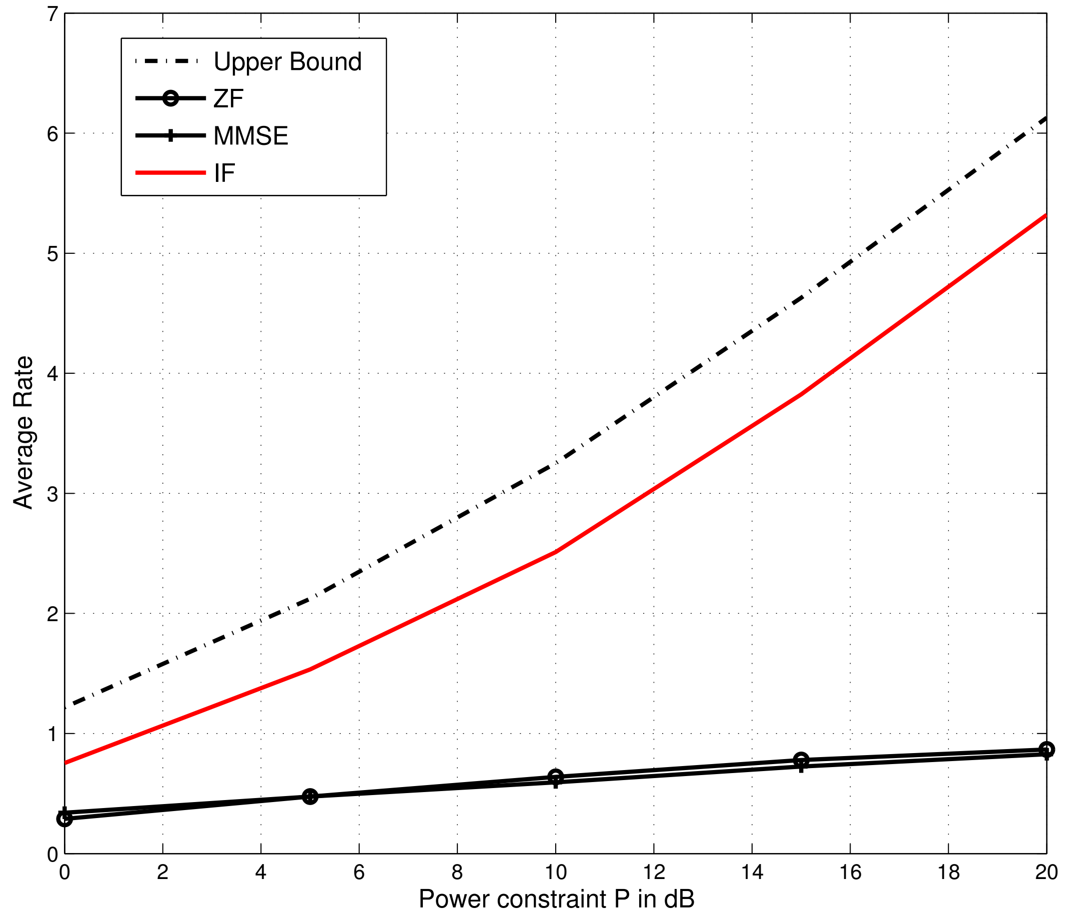


Figure 2: Average rate comparison with $L = N = 2$

Experimental Studies

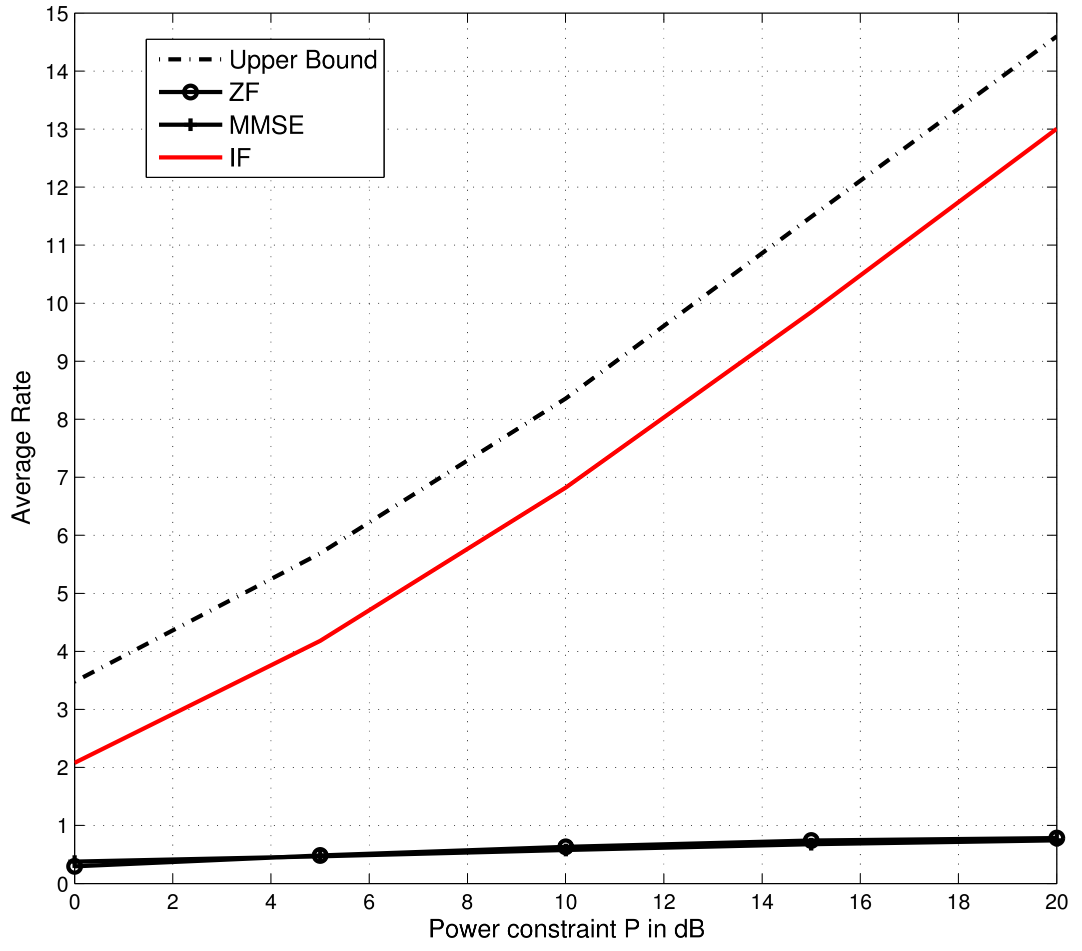


Figure 3: Average rate comparison with $L = N = 4$

Conclusions

- In this work, we consider IF linear receiver design with respect to the channel conditions over MIMO channels where each transmit antenna delivers an independent data stream.
- We present algorithms to design the IF full rank coefficient matrix with integer elements, such that the total achievable rate is maximized, based on Fincke-Pohst method.
- First, we will generate feasible candidate integer vector set instead of the whole integer searching space based on Fincke-Pohst method.
- Then we try to pick up integer vectors within the searching set to construct the full rank IF coefficient matrix, while in the meantime, the total achievable rate is maximized.
- Numerical studies show the comparisons of other traditional linear receivers.

Thank You!