2014 Sino-German Workshop
Bridging Theory and Practice in Wireless Communications and Networking

Shenzhen Research Institute, The Chinese University of Hong Kong
March 4-7, 2014

Random Access and Codes on Graphs: From Theory to Practice

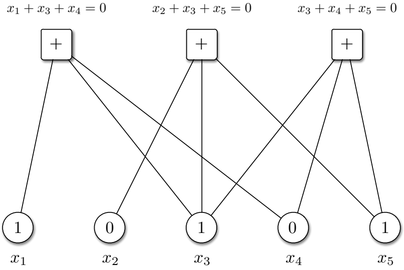
Gianluigi Liva, gianluigi.liva@dlr.de

Institute for Communications and Navigation
German Aerospace Center, DLR

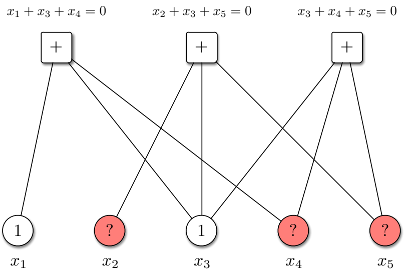
Outline

1. Coded Slotted Aloha
2. Throughput Analysis
3. From Theory to Practice
4. Conclusions

Prologue
Erasure Correcting Codes

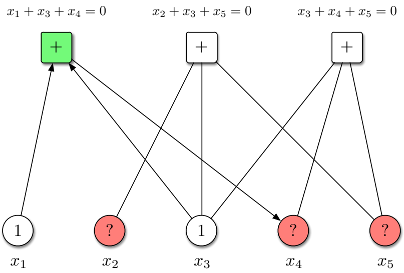


Prologue
Erasure Correcting Codes

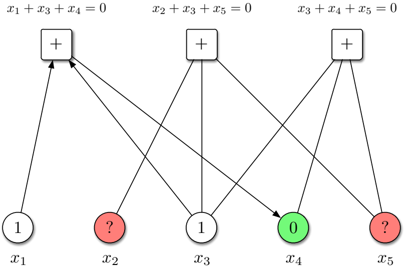


DLR

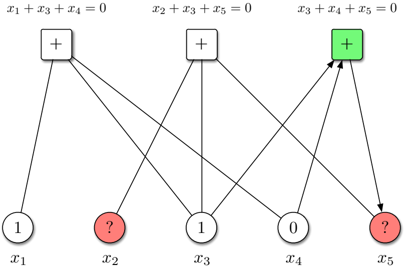
Prologue
Erasure Correcting Codes



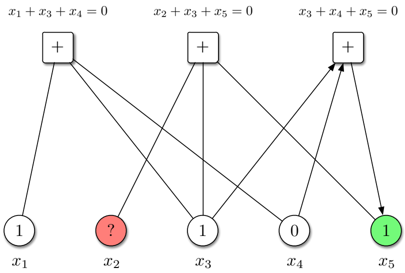
Prologue
Erasure Correcting Codes



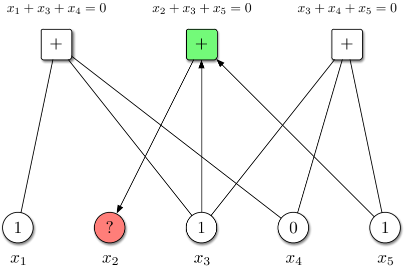
Prologue
Erasure Correcting Codes



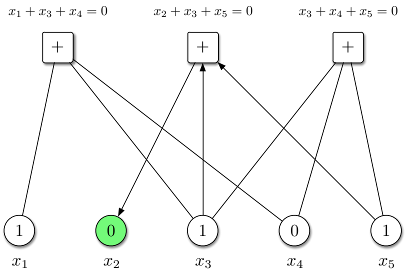
Prologue
Erasure Correcting Codes



Prologue
Erasure Correcting Codes



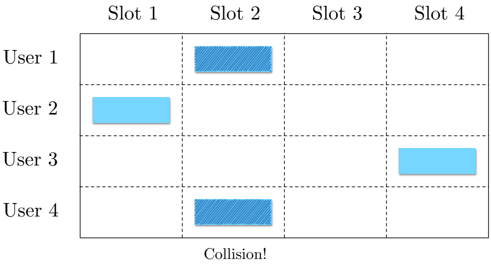
Prologue
Erasure Correcting Codes



Prologue
Multiple Access: Slotted Aloha (Framed)

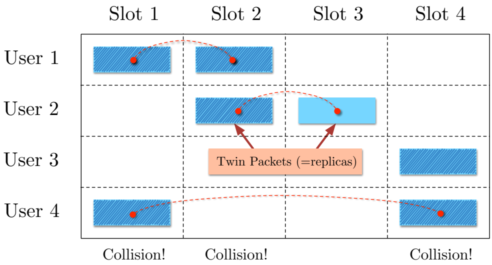
MAC Frame

Prologue
Multiple Access: Slotted Aloha (Framed)



Collision $\equiv$ Erasure

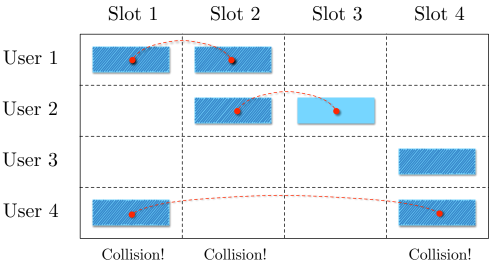
Prologue
Multiple Access: Coded Slotted Aloha (Framed)



Collision $\equiv$ Erasure

Packet Copies $\equiv$ Repetition Code

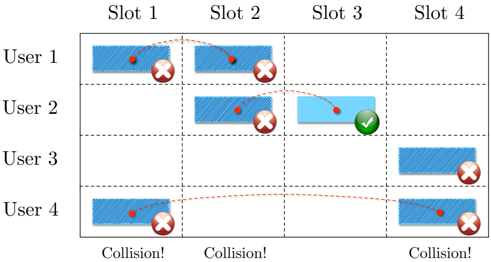
Prologue
Multiple Access: Coded Slotted Aloha (Framed)



Collision ≡ Erasure

Packet Copies ≡ Repetition Code

Prologue
Multiple Access: Coded Slotted Aloha (Framed)

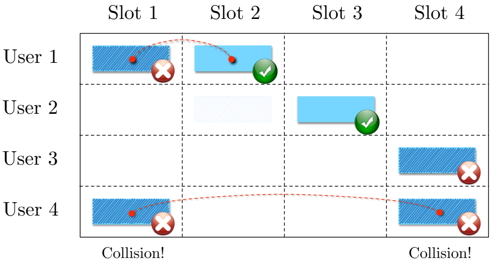


Successive Interference Cancellation

$\equiv$

Iterative Erasure Decoding

Prologue
Multiple Access: Coded Slotted Aloha (Framed)

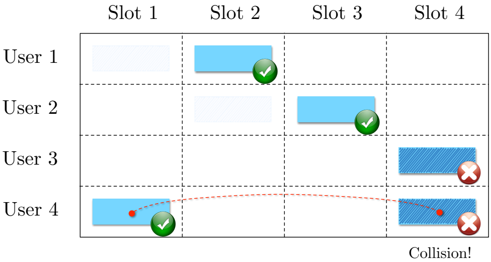


Successive Interference Cancellation

$\equiv$

Iterative Erasure Decoding

Prologue
Multiple Access: Coded Slotted Aloha (Framed)

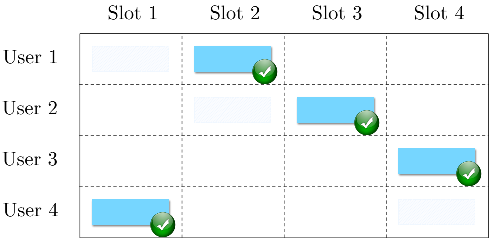


Successive Interference Cancellation

$\equiv$

Iterative Erasure Decoding

Prologue
Multiple Access: Coded Slotted Aloha (Framed)



Successive Interference Cancellation

≡

Iterative Erasure Decoding

Outline

1. Coded Slotted Aloha
2. Throughput Analysis
3. From Theory to Practice
4. Conclusions

Coded Slotted Aloha
Overview

- Slotted Aloha (SA) [Abramson1970]: Adopted as the initial access scheme in both cellular terrestrial and satellite networks
- Diversity slotted Aloha (DSA) [Choudhury1983]: Packet repetition (twin replicas) to achieve a slight throughput enhancement at low loads
- Contention resolution diversity slotted Aloha (CRDSA) [Casini2007]: A more efficient use of the packet repetition

[Abramson1970] N. Abramson, “The Aloha system - another alternative for computer communications,” in *Proc. of 1970 Fall Joint Computer Conf.*, 1970.

[Choudhury1983] G. Choudhury and S. Rappaport, “Diversity Aloha - a random access scheme for satellite communications,” *IEEE Trans. Commun.*, Mar. 1983.

[Casini2007] E. Casini, R. D. Gaudenzi, and O. del Rio Herrero, “Contention resolution diversity slotted Aloha (CRDSA): An enhanced random access scheme for satellite access packet networks.,” *IEEE Trans.Wireless Commun.*, Apr. 2007.

Coded Slotted Aloha
Overview

- Slotted Aloha (SA) [Abramson1970]: Adopted as the initial access scheme in both cellular terrestrial and satellite networks
- Diversity slotted Aloha (DSA) [Choudhury1983]: Packet repetition (twin replicas) to achieve a slight throughput enhancement at low loads
- Contention resolution diversity slotted Aloha (CRDSA) [Casini2007]: A more efficient use of the packet repetition

[Abramson1970] N. Abramson, “The Aloha system - another alternative for computer communications,” in *Proc. of 1970 Fall Joint Computer Conf.*, 1970.

[Choudhury1983] G. Choudhury and S. Rappaport, “Diversity Aloha - a random access scheme for satellite communications,” *IEEE Trans. Commun.*, Mar. 1983.

[Casini2007] E. Casini, R. D. Gaudenzi, and O. del Rio Herrero, “Contention resolution diversity slotted Aloha (CRDSA): An enhanced random access scheme for satellite access packet networks.,” *IEEE Trans.Wireless Commun.*, Apr. 2007.

Coded Slotted Aloha
Overview

- Slotted Aloha (SA) [Abramson1970]: Adopted as the initial access scheme in both cellular terrestrial and satellite networks
- Diversity slotted Aloha (DSA) [Choudhury1983]: Packet repetition (twin replicas) to achieve a slight throughput enhancement at low loads
- Contention resolution diversity slotted Aloha (CRDSA) [Casini2007]: A more efficient use of the packet repetition

---

[Abramson1970] N. Abramson, "The Aloha system - another alternative for computer communications," in *Proc. of 1970 Fall Joint Computer Conf.*, 1970.

[Choudhury1983] G. Choudhury and S. Rappaport, "Diversity Aloha - a random access scheme for satellite communications," *IEEE Trans. Commun.*, Mar. 1983.

[Casini2007] E. Casini, R. D. Gaudenzi, and O. del Rio Herrero, "Contention resolution diversity slotted Aloha (CRDSA): An enhanced random access scheme for satellite access packet networks.," *IEEE Trans.Wireless Commun.*, Apr. 2007.

Coded Slotted Aloha
Overview

- Irregular repetition slotted Aloha (IRSA) [Liva2011]: Equivalence with codes on graphs
- Overall framework: Coded Slotted Aloha (CSA) [Paolini2011]
- IRSA achieves a throughput of 1 [packet/slot] [Narayanan2012]

[Liva2011] G. Liva, "Graph-based analysis and optimization of contention resolution diversity slotted Aloha," *IEEE Trans. Commun.*, Feb. 2011.

[Paolini2011] E. Paolini, G. Liva, and M. Chiani, "High Throughput Random Access via Codes on Graphs: Coded Slotted Aloha," IEEE ICC 2011.

[Narayanan2012] K.R. Narayanan and H.F. Pfister "Iterative collision resolution for slotted Aloha: An optimal uncoordinated transmission policy," 2012 7th International Symposium on Turbo Codes and Iterative Information Processing. Aug. 2012.

Coded Slotted Aloha
Overview

- Irregular repetition slotted Aloha (IRSA) [Liva2011]: Equivalence with codes on graphs
- Overall framework: Coded Slotted Aloha (CSA) [Paolini2011]
- IRSA achieves a throughput of 1 [packet/slot] [Narayanan2012]

[Liva2011] G. Liva, "Graph-based analysis and optimization of contention resolution diversity slotted Aloha," *IEEE Trans. Commun.*, Feb. 2011.

[Paolini2011] E. Paolini, G. Liva, and M. Chiani, "High Throughput Random Access via Codes on Graphs: Coded Slotted Aloha," IEEE ICC 2011.

[Narayanan2012] K.R. Narayanan and H.F. Pfister "Iterative collision resolution for slotted Aloha: An optimal uncoordinated transmission policy," 2012 7th International Symposium on Turbo Codes and Iterative Information Processing. Aug. 2012.

Coded Slotted Aloha
Overview

- Irregular repetition slotted Aloha (IRSA) [Liva2011]: Equivalence with codes on graphs
- Overall framework: Coded Slotted Aloha (CSA) [Paolini2011]
- IRSA achieves a throughput of 1 [packet/slot] [Narayanan2012]

[Liva2011] G. Liva, “Graph-based analysis and optimization of contention resolution diversity slotted Aloha,” *IEEE Trans. Commun.*, Feb. 2011.

[Paolini2011] E. Paolini, G. Liva, and M. Chiani, “High Throughput Random Access via Codes on Graphs: Coded Slotted Aloha,” IEEE ICC 2011.

[Narayanan2012] K.R. Narayanan and H.F. Pfister “Iterative collision resolution for slotted Aloha: An optimal uncoordinated transmission policy,” 2012 7th International Symposium on Turbo Codes and Iterative Information Processing. Aug. 2012.

Coded Slotted Aloha
Related Works

- Frame-less Aloha with Successive Interference Cancellation (SIC) [Stefanovic2014]
- IRSA with Multi-User Detection [Ghanbarinejad2013]
- SIC Tree Algorithm [Yu2007]
- Zig-Zag and Sig-Sag decoding [Gollakota2008][Tehrani2011]

[Stefanovic2014] C. Stefanovic, P. Popovski, “Aloha Random Access that Operates as a Rateless Code,” *IEEE Trans. Commun.*, Feb. 2014.

[Ghanbarinejad2013] M. Ghanbarinejad and C. Schlegel, “Irregular repetition slotted Aloha with multiuser detection,” in Proc. the 10th Annual Conf. Wireless On-demand Netw. Syst. Services, Mar. 2013.

[Yu2007] Y. Yu and G. B. Giannakis,“High-throughput random access using successive interference cancellation in a tree algorithm,” *IEEE Trans. Inf. Theory*, Dec. 2007.

[Gollakota2008] S. Gollakota and D. Katabi, “Zigzag decoding: combating hidden terminals in wireless networks,” in Proc. SIGCOMM, 2008.

[Tehrani2011] A. Tehrani, A. Dimakis, and M. Neely, “Sigsag: Iterative detection through soft message-passing,” *IEEE Trans. Signal Process.*, Dec. 2011.

Coded Slotted Aloha
Related Works

- Frame-less Aloha with Successive Interference Cancellation (SIC) [Stefanovic2014]
- IRSA with Multi-User Detection [Ghanbarinejad2013]
- SIC Tree Algorithm [Yu2007]
- Zig-Zag and Sig-Sag decoding [Gollakota2008][Tehrani2011]

[Stefanovic2014] C. Stefanovic, P. Popovski, "Aloha Random Access that Operates as a Rateless Code," *IEEE Trans. Commun.*, Feb. 2014.

[Ghanbarinejad2013] M. Ghanbarinejad and C. Schlegel, "Irregular repetition slotted Aloha with multiuser detection," in Proc. the 10th Annual Conf. Wireless On-demand Netw. Syst. Services, Mar. 2013.

[Yu2007] Y. Yu and G. B. Giannakis,"High-throughput random access using successive interference cancellation in a tree algorithm," *IEEE Trans. Inf. Theory*, Dec. 2007.

[Gollakota2008] S. Gollakota and D. Katabi, "Zigzag decoding: combating hidden terminals in wireless networks," in Proc. SIGCOMM, 2008.

[Tehrani2011] A. Tehrani, A. Dimakis, and M. Neely, "Sigsag: Iterative detection through soft message-passing," *IEEE Trans. Signal Process.*, Dec. 2011.

Coded Slotted Aloha
Related Works

- Frame-less Aloha with Successive Interference Cancellation (SIC) [Stefanovic2014]
- IRSA with Multi-User Detection [Ghanbarinejad2013]
- SIC Tree Algorithm [Yu2007]
- Zig-Zag and Sig-Sag decoding [Gollakota2008][Tehrani2011]

[Stefanovic2014] C. Stefanovic, P. Popovski, “Aloha Random Access that Operates as a Rateless Code,” *IEEE Trans. Commun.*, Feb. 2014.

[Ghanbarinejad2013] M. Ghanbarinejad and C. Schlegel, “Irregular repetition slotted Aloha with multiuser detection,” in Proc. the 10th Annual Conf. Wireless On-demand Netw. Syst. Services, Mar. 2013.

[Yu2007] Y. Yu and G. B. Giannakis,“High-throughput random access using successive interference cancellation in a tree algorithm,” *IEEE Trans. Inf. Theory*, Dec. 2007.

[Gollakota2008] S. Gollakota and D. Katabi, “Zigzag decoding: combating hidden terminals in wireless networks,” in Proc. SIGCOMM, 2008.

[Tehrani2011] A. Tehrani, A. Dimakis, and M. Neely, “Sigsag: Iterative detection through soft message-passing,” *IEEE Trans. Signal Process.*, Dec. 2011.

Coded Slotted Aloha
Related Works

- Frame-less Aloha with Successive Interference Cancellation (SIC) [Stefanovic2014]
- IRSA with Multi-User Detection [Ghanbarinejad2013]
- SIC Tree Algorithm [Yu2007]
- Zig-Zag and Sig-Sag decoding [Gollakota2008][Tehrani2011]

[Stefanovic2014] C. Stefanovic, P. Popovski, “Aloha Random Access that Operates as a Rateless Code,” *IEEE Trans. Commun.*, Feb. 2014.

[Ghanbarinejad2013] M. Ghanbarinejad and C. Schlegel, “Irregular repetition slotted Aloha with multiuser detection,” in Proc. the 10th Annual Conf. Wireless On-demand Netw. Syst. Services, Mar. 2013.

[Yu2007] Y. Yu and G. B. Giannakis,“High-throughput random access using successive interference cancellation in a tree algorithm,” *IEEE Trans. Inf. Theory*, Dec. 2007.

[Gollakota2008] S. Gollakota and D. Katabi, “Zigzag decoding: combating hidden terminals in wireless networks,” in Proc. SIGCOMM, 2008.

[Tehrani2011] A. Tehrani, A. Dimakis, and M. Neely, “Sigsag: Iterative detection through soft message-passing,” *IEEE Trans. Signal Process.*, Dec. 2011.

Coded Slotted Aloha
System Model (Repetition Codes)

a. Collisions are destructive

b. In absence of interference, packets are decoded with high probability

c. Packet replicas have a pointer to the their respective copies

d. If packet is successfully decoded, the pointer is extracted and the interference contributions caused by the replicas on the corresponding slots are removed

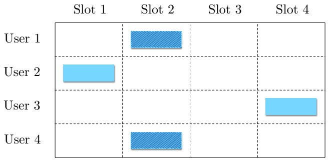
e. The procedure is iterated until no more clean packets are discovered

Coded Slotted Aloha

System Model (Repetition Codes)

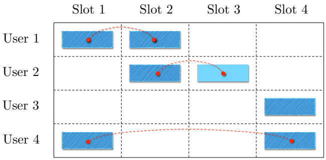
- M users, each attempting one packet transmission within a frame
- Number of slots N_{SA}
- Load given by

$$G = \frac{M}{N_{\text{SA}}}$$



Slotted Aloha

$G = 1$ [packets/slot]

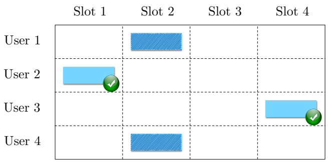


Coded Slotted Aloha

$G = 1$ [packets/slot]

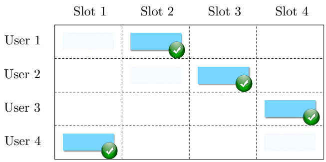
Coded Slotted Aloha
System Model (Repetition Codes)

- T is the throughput in terms of successful packet transmissions per slot
- Replicas shall not be counted...



Slotted Aloha

$T = 0.5$ [packets/slot]

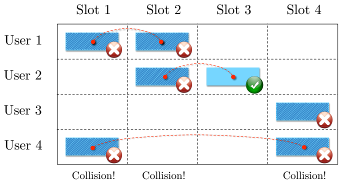


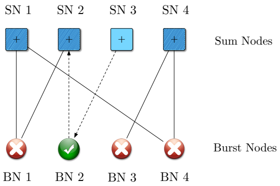
Coded Slotted Aloha

$T = 1$ [packets/slot]

Coded Slotted Aloha
Irregular Repetition Slotted Aloha (IRSA)

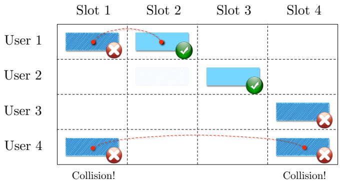
- Bipartite graph representation
  - slots ↔ sum (slot) nodes
  - packets ↔ burst (packet) nodes
  - replicas ↔ edges

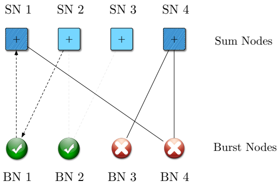




Coded Slotted Aloha
Irregular Repetition Slotted Aloha(IRSA)

- Bipartite graph representation
  - slots ↔ sum (slot) nodes
  - packets ↔ burst (packet) nodes
  - replicas ↔ edges

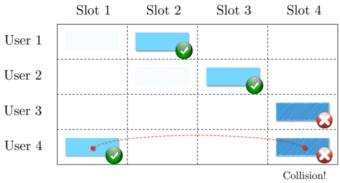


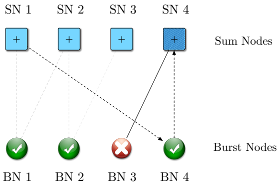


Coded Slotted Aloha

Irregular Repetition Slotted Aloha (IRSA)

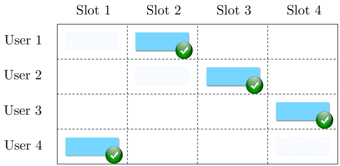
- Bipartite graph representation
  - slots ↔ sum (slot) nodes
  - packets ↔ burst (packet) nodes
  - replicas ↔ edges

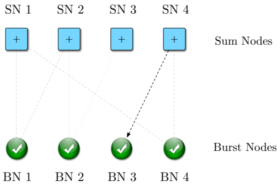




Coded Slotted Aloha
Irregular Repetition Slotted Aloha (IRSA)

- Bipartite graph representation
  - slots ↔ sum (slot) nodes
  - packets ↔ burst (packet) nodes
  - replicas ↔ edges





Coded Slotted Aloha
General Case

- Combines *time-hopping multiple access* (THMA) [Lam1990] and successive interference cancellation (SIC).
- Each user divides the packet in k slices
- Slices encoded by an (n_h, k) erasure correcting code $\mathcal{C}_h$.
- The code $\mathcal{C}_h$ is picked randomly from a set $\mathcal{C} = \{\mathcal{C}_1, \ldots, \mathcal{C}_{n_c}\}$ of component codes, all with the same dimension k.
- Encoded slices transmitted in n_h slots picked at random.

[Lam1990] A. Lam and D. Sarwate, "Time-Hopping and Frequency-Hopping Multiple-Access Packet Communications," *IEEE Trans. Commun.*, vol. 38, pp. 875–888, June 1990.

Coded Slotted Aloha
General Case

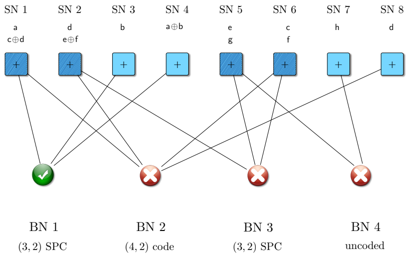
- The code $\mathcal{C}_h$ is picked with probability P_h
- Rate of the scheme:

$$R = \frac{k}{\sum_{h=1}^{n_c} P_h n_h} = \frac{k}{\bar{n}}$$

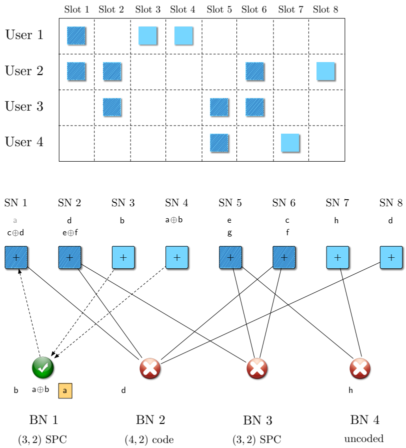
- For a fixed frame duration, the frame is composed of $N_{\text{CSA}} = kN_{\text{SA}}$ slots
- Iterative SIC process is combined with local MAP erasure decoding

Coded Slotted Aloha
Example

	Slot 1	Slot 2	Slot 3	Slot 4	Slot 5	Slot 6	Slot 7	Slot 8
User 1	a		b	a⊕b				
User 2	c⊕d	d				c		d
User 3		e⊕f			e	f		
User 4					g		h	

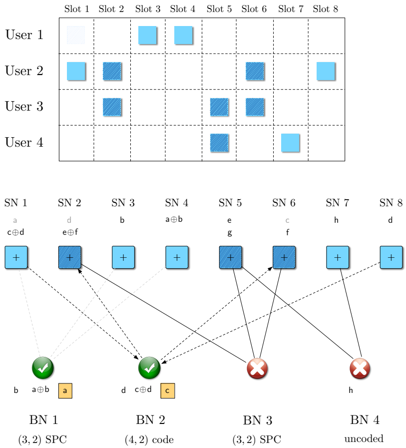


Coded Slotted Aloha
Example

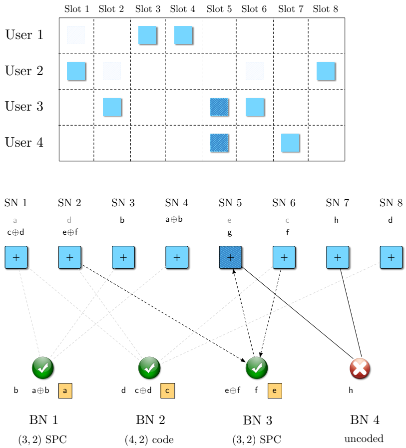


Coded Slotted Aloha

Example

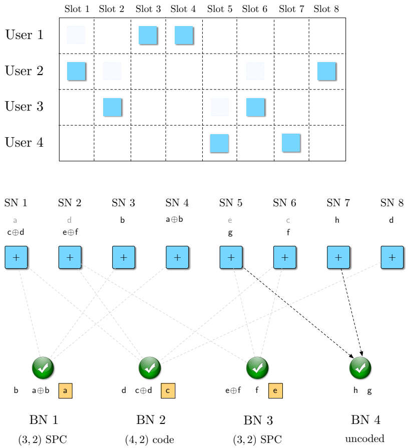


Coded Slotted Aloha
Example



DLR

Coded Slotted Aloha
Example



Asymptotic SIC Analysis
Density Evolution

- Asymptotic setting:
  - $N_{\mathsf{SA}} = N_{\mathsf{CSA}}/k \to \infty$
  - $M = G \cdot N_{\mathsf{SA}} \to \infty$
- Analyze the behavior of iterative SIC with density evolution (well-established analysis tool in the field of modern coding theory)

Asymptotic SIC Analysis
Density Evolution

Threshold phenomenon

For a given $\mathcal{C} = \{\mathcal{C}_1, \ldots, \mathcal{C}_{n_c}\}$ and a given $\boldsymbol{P} = \{P_h\}_{h=1,\ldots,n_c}$ there exists $G^*(\mathcal{C}, \boldsymbol{P})$ s.t.

- for all $0 < G < G^*(\mathcal{C}, \boldsymbol{P})$, the residual packet erasure probability tends to zero as the number of IC iterations tends to infinity
- for all $G > G^*(\mathcal{C}, \boldsymbol{P})$, decoding fails with a probability always bounded away from 0

- The asymptotic threshold G^* depends on the component codes and on their probabilities
- Look for $\mathcal{C}$ and $\boldsymbol{P}$ leading large thresholds, allowing transmissions with vanishing error probability for any load $G < G^*(\mathcal{C}, \boldsymbol{P})$

Asymptotic SIC Analysis
Density Evolution

Threshold phenomenon

For a given $\mathcal{C} = \{\mathcal{C}_1, \ldots, \mathcal{C}_{n_c}\}$ and a given $\boldsymbol{P} = \{P_h\}_{h=1,\ldots,n_c}$ there exists $G^*(\mathcal{C}, \boldsymbol{P})$ s.t.

- for all $0 < G < G^*(\mathcal{C}, \boldsymbol{P})$, the residual packet erasure probability tends to zero as the number of IC iterations tends to infinity
- for all $G > G^*(\mathcal{C}, \boldsymbol{P})$, decoding fails with a probability always bounded away from 0

- The asymptotic threshold G^* depends on the component codes and on their probabilities
- Look for $\mathcal{C}$ and $\boldsymbol{P}$ leading large thresholds, allowing transmissions with vanishing error probability for any load $G < G^*(\mathcal{C}, \boldsymbol{P})$

Asymptotic SIC Analysis
Density Evolution

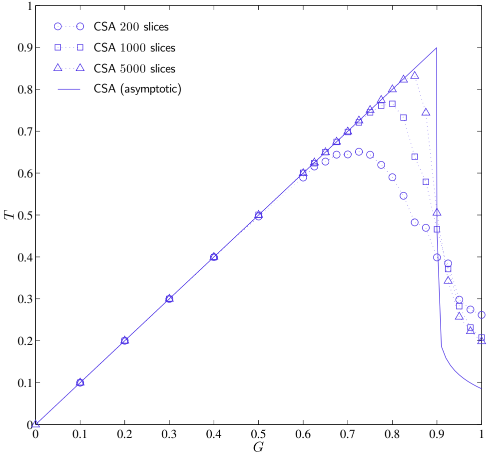
Threshold phenomenon

For a given $\mathcal{C} = \{\mathcal{C}_1, \ldots, \mathcal{C}_{n_c}\}$ and a given $\boldsymbol{P}= \{P_h\}_{h=1,\ldots,n_c}$ there exists $G^*(\mathcal{C}, \boldsymbol{P})$ s.t.

- for all $0 < G < G^*(\mathcal{C}, \boldsymbol{P})$, the residual packet erasure probability tends to zero as the number of IC iterations tends to infinity
- for all $G > G^*(\mathcal{C}, \boldsymbol{P})$, decoding fails with a probability always bounded away from 0

- The asymptotic threshold G^* depends on the component codes and on their probabilities
- Look for $\mathcal{C}$ and $\boldsymbol{P}$ leading large thresholds, allowing transmissions with vanishing error probability for any load $G < G^*(\mathcal{C}, \boldsymbol{P})$

Asymptotic SIC Analysis
Density Evolution



Density Evolution Equations

- At the ℓ-th IC iteration, let
  - p_ℓ be the average message erasure probability from the SNs to the BNs
  - q_ℓ be the average message erasure probability from the BNs to the SNs

$$q_\ell = \frac{1}{\bar{n}} \sum_{h=1}^{n_c} P_h \sum_{t=0}^{n_h-1} p_{\ell-1}^t (1-p_{\ell-1})^{n_h-1-t} \left[(n_h - t)\tilde{e}^{(h)}_{n_h-t} - (t+1)\tilde{e}^{(h)}_{n_h-1-t} \right]$$

$$p_\ell = 1 - \exp\left(-\frac{G}{R} q_\ell \right)$$

where $\tilde{e}^{(h)}_g$ are the component codes information functions

$$G^*(\mathcal{C}, \boldsymbol{P}) = \sup\{G \geq 0 : p_\ell \to 0 \text{ as } \ell \to \infty,\ p_0 = 1\}$$

Outline

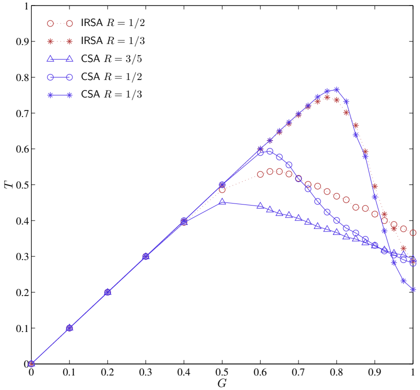
Threshold Optimization

- Distribution profiles $\boldsymbol{P}$ and corresponding thresholds $G^*(\boldsymbol{P})$ reported for optimized IRSA and CSA (with $k = 2$) schemes under the random code hypothesis

IRSA							G^*
R	$(2,1)$	$(3,1)$	$(6,1)$				
$1/3$	0.55401	0.26131	0.18467				0.879
$2/5$	0.62241	0.25517	0.12241				0.782
$1/2$	1.00000						0.500
CSA, $k = 2$							G^*
R	$(3,2)$	$(4,2)$	$(5,2)$	$(8,2)$	$(9,2)$	$(12,2)$	
$1/3$	0.08845	0.54418	0.12149			0.24587	0.868
$2/5$	0.15305	0.48508	0.13549	0.11423	0.11212		0.797
$1/2$		1.00000					0.656
$3/5$	0.66667	0.33333					0.409

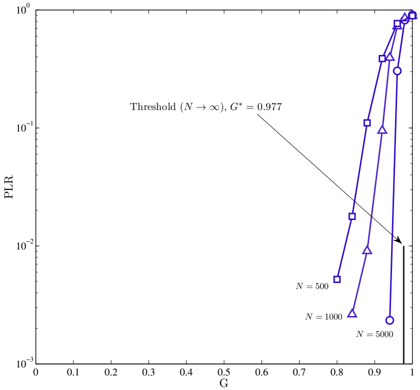
DLR

Throughput Analysis for Optimized Profiles



- $N_{\mathsf{SA}} = 500$, $N_{\mathsf{CSA}} = 1000$
- Specific choice of linear block codes in the set $\mathcal{C}$
- 6 codes, all with $k = 2$, and $n \in \{4, 5, 8, 9, 12\}$

Coded Slotted Aloha without Feedback Channel



- Packet Loss Rate for Coded SA based on optimized profiles
- $N_{\text{SA}} = 5000, 1000, 500$, maximum iteration count set to 100
- Throughput close to 1 packet/frame without feedback channel - no retransmissions!!!

How Far Can We Push $G^*(\mathcal{C}, \boldsymbol{P})$ for given R?

Theorem

For rational R and $0 < R \leq 1$, let $\mathbb{G}(R)$ be the unique positive solution to the equation

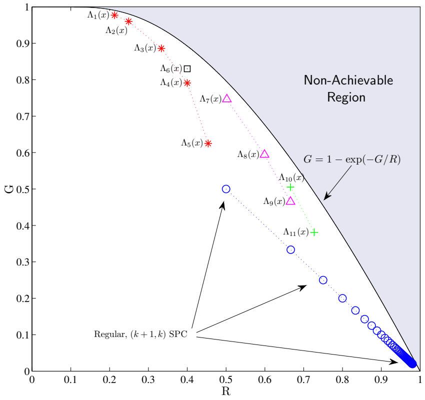
$$G = 1 - e^{-G/R}$$

in $[0, 1)$. Then, the threshold $G^*(\mathcal{C}, \boldsymbol{P})$ fulfills

$$G^*(\mathcal{C}, \boldsymbol{P}) < \mathbb{G}(R)$$

for *any* choice of $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{n_c}\}$ and $\boldsymbol{P}$ associated with a rate R

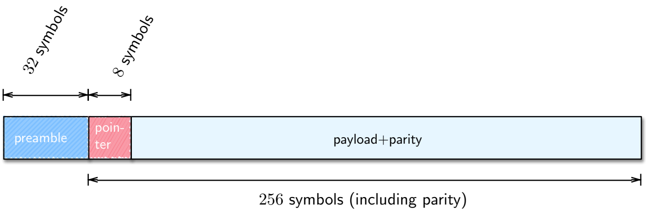
How Far Can We Push $G^*(\mathcal{C}, P)$ for given R?



Outline

A Closer Look at Successive Interference Cancellation
Packet Format

- 32-symbols preamble for detection and initial channel estimation
- 256-bit payload including 16-bits header for replica pointer
- 256 parity bits
- QPSK, square-root raised cosine matched filter (MF), roll-off 0.2



A Closer Look at Successive Interference Cancellation
Signal Processing

- Perfect power control
- l users attempt transmission within the same slot
- Complex baseband signal transmitted by the i-th user

$$u^{(i)}(t) = \sum_{v=1}^{N_s} b_v^{(i)} \gamma(t - vT_s)$$

where N_s is the number of symbols per segment, $\{b_v^{(i)}\}$ is the sequence of symbols and T_s is the symbol period

- Pulse shape $\gamma(t) = \mathcal{F}^{-1}\left\{\sqrt{\mathrm{RC}(f)}\right\}$, where $\mathrm{RC}(f)$ the frequency response of the MF

A Closer Look at Successive Interference Cancellation
Signal Processing

- Each contribution is received with a random delay ϵ_i, a random frequency offset $f_i \sim \mathcal{U}[-f_{\max}, f_{\max}]$ and a random phase offset $\phi_i \sim \mathcal{U}[0, 2\pi)$
- After the MF,

$$r(t) = \sum_{i=1}^{l} z^{(i)}(t) * h(t) + n(t)$$

where $n(t)$ is the Gaussian noise contribution, $h(t) = \gamma^*(-t)$ is the MF impulse response and

$$z^{(i)}(t) = \sum_{v=1}^{N_s} b_v^{(i)} \gamma(t - vT_s - \epsilon_i) \exp(j2\pi f_i t + j\phi_i)$$

A Closer Look at Successive Interference Cancellation
Signal Processing: Assumption

- Frequency shifts that are small w.r.t. the signal bandwidth (i.e., $f_{\max} T_s \ll 1$). Thus

$$r(t) \approx \sum_{i=1}^{l} \tilde{u}^{(i)}(t - \epsilon_i) e^{j2\pi f_i t + j\phi_i} + n(t)$$

  where $\tilde{u}^{(i)}(t)$ is the response of the MF to $u^{(i)}(t)$
- $\tilde{u}^{(1)}(t)$ ss the useful term and $\tilde{u}^{(2)}(t), \tilde{u}^{(3)}(t), \ldots, \tilde{u}^{(l)}(t)$ are the interference contributions to be cancelled
- First, estimate the set of parameters $\{\epsilon_i, f_i, \phi_i\}$, for $i \in \{2, \ldots, l\}$

A Closer Look at Successive Interference Cancellation
Signal Processing: Assumption

- Typically, in satellite applications ϵ_i and f_i can be accurately estimated on the recovered replicas (i.e., their values remain constant through the frame)
- ϕ_i, which may not be stable from a slot to slot.
- Recall that the symbol sequences $\{b_v^{(i)}\}$ (for $i \in \{2 \ldots l\}$) are known at the receiver

A Closer Look at Successive Interference Cancellation

- Denote by $y^{(i)}(t)$ the signal at the input of the phase estimator for the i-th contribution
- In the first step, the input signal is given by $y^{(2)}(t) = r(t)$ and the phase of the first interfering user is estimated as

$$\hat{\phi}_2 = \arg\left\{\sum_{v=1}^{N_s} y_v^{(2)} \left(b_v^{(2)}\right)^*\right\}$$

with

$$y_v^{(2)} = y^{(2)}(vT_s + \epsilon_2)e^{-j2\pi f_2(vT_s+\epsilon_2)}.$$

- After the estimation of the phase offset for the first interferer, the corresponding signal can be reconstructed as $\tilde{u}^{(2)}(t-\epsilon_2)e^{j2\pi f_2 t + j\hat{\phi}_2}$ and its contribution can be removed, i.e.

$$y^{(3)}(t) = y^{(2)}(t) - \tilde{u}^{(2)}(t-\epsilon_2)e^{j2\pi f_2 t + j\hat{\phi}_2}.$$

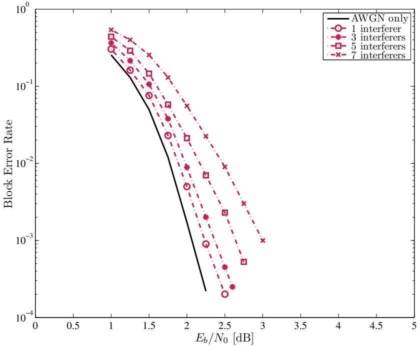
A Closer Look at Successive Interference Cancellation

- The SIC proceeds serially
- After the cancellation of the $l-1$ contributions the residual signal, denoted by $y^{(1)}(t)$, is given by the 1-st user's contribution, the noise $n(t)$, and a residual interference term $\nu(t)$ due to the imperfect estimation of the interferers' phases (causing imperfect SIC), i.e.,

$$y^{(1)}(t) = \tilde{u}^{(1)}(t-\epsilon_1)e^{j2\pi f_1 t + j\phi_1} + n(t) + \nu(t)$$

DLR

A Closer Look at Successive Interference Cancellation



- LDPC code over $\mathbb{F}_{256}$
- $k = 256$ bits and $n = 512$ bits
- The users are coarsely synchronized
- $f_i \sim \mathcal{U}\left[-f_{max}, +f_{max}\right]$
- $f_{max} = 0.01 \times B_s$, being B_s the symbol rate
- Random phase offset for each replica
- The modulation is QPSK

Coded Slotted Aloha in Practice

- The repetition-based variant of CSA is the random access method adopted by the 2nd generation of the Digital Video Broadcasting (DVB) Return Channel via Satellite (RCS) standard for interactive satellite services
- DLR owns a SDR (ETTUS)-GPU-based gateway (implementing a similar random access scheme employing spreading in addition, ETSI S-MIM standard) working at 10 Mbps
- We have developed together with TUM a multi-user detector on a SDR-GPU platform, which may further enhance the performance enhanced random access protocols (~~destructive collisions~~)

Outline

Conclusion

- Coded Slotted Aloha expresses most of its potential on a collision channel without feedback, thanks to its high reliability
- Shares several aspects with LDPC codes (and their generalization) over erasure channels
- Analogy:

$$\text{FEC} \Leftrightarrow \text{ARQ}$$
$$\text{CSA} \Leftrightarrow \text{SA}$$

- The CSA graph-based random access scheme can approach an efficiency of 1 packet/slot without retransmissions

Reference

- G. Liva, "Graph-Based Analysis and Optimization of Contention Resolution Diversity Slotted Aloha," IEEE Trans. Commun., Feb., 2011.
- E. Paolini, G. Liva, and M. Chiani, "High Throughput Random Access via Codes on Graphs: Coded Slotted Aloha," IEEE ICC 2011.
- E. Paolini, G. Liva, M. Chiani, "Graph-Based Random Access for the Collision Channel without Feed-Back: Capacity Bound," IEEE Globecom 2011.
- G. Liva, E. Paolini, M. Lentmaier, M.Chiani,"Spatially-Coupled Random Access on Graphs," IEEE ISIT 2012.
- E. Paolini, G. Liva, M. Chiani, "Coded Slotted Aloha: A Graph-Based Method for Uncoordinated Multiple Access," submitted to IEEE Trans. Inf. Theory, available on Arxiv.org.

DLR

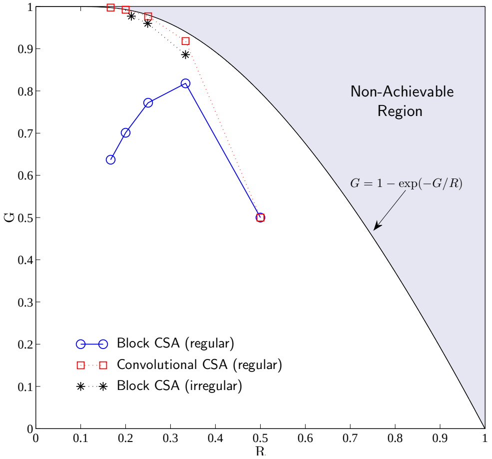
谢谢

Thank you!

Approaching the Upper Bound: Spatially Coupled CSA

- Idea: Exploit the spatial coupling effect within the CSA framework to improve the threshold.
- Consider the case where all users adopt the same repetition code (of length d) for each transmission.
- Observe a threshold saturation effect also for spatially coupled CSA.

Block and Convolutional CSA Schemes Performance Comparison



Threshold Saturation Effect

- Genie-Aided MAP Decoding: The bipartite graph is revealed to the decoder by a genie, which enables MAP erasure decoding at the gateway.
- We compare the thresholds under
  - Block regular CSA ($G^{\text{IT}}_{\text{block}}$)
  - Convolutional CSA ($G^{\text{IT}}_{\text{conv}}$)
  - Genie-aided decoding ($\overline{G}^{\text{MAP}}_{\text{block}}$)
  - Upper bound (G^*)

d	$G^{\text{IT}}_{\text{block}}$	$G^{\text{IT}}_{\text{conv}}$	$\overline{G}^{\text{MAP}}_{\text{block}}$	G^*
2	0.5	0.5	0.5	0.7969
3	0.8184	0.9179	0.9179	0.9405
4	0.7722	0.9767	0.9767	0.9802
5	0.7017	0.9924	0.9924	0.9931

[Kudekar2011] S. Kudekar, T. Richardson, R. Urbanke, “Threshold saturation via spatial coupling: Why convolutional LDPC ensembles perform so well over the BEC,” *IEEE Trans. IT*, Feb. 2011.