JACOBS UNIVERSITY

Outage-free Transmit Power Minimization with Imperfect CSIT

The 2014 Sino-Germany Workshop

"Bridging Theory and Practice in Wireless Communications and Networking"

Giuseppe Abreu

g.abreu@jacobs-university.de

School of Engineering and Sciences
Jacobs University Bremen

March 4, 2014

Evolution of Cellular Technology

Bridging Theory and Practice in Cellular Systems

	GSM	3G	4G	5G
Application	12 kbps	1 Mbps	10 Mbps	1 Gbps
Access	20 kbps	24 kbps	300 Mbps	10 Gbps
Latency	150 ms	50 ms	10 ms	1 ms
Switching Time	Few seconds	500 ms	200 ms	10 ms

Evolution of Cellular Technology

Bridging Theory and Practice in Cellular Systems

	GSM	3G	4G	5G
Application	12 kbps	1 Mbps	10 Mbps	1 Gbps
Access	20 kbps	24 kbps	300 Mbps	10 Gbps
Latency	150 ms	50 ms	10 ms	1 ms
Switching Time	Few seconds	500 ms	200 ms	10 ms

Trend: 1000$\times$ traffic in 10 years!

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)
  - 3G - More spectrum

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ “Digital” (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ “Digital” (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more...

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ “Digital” (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** ($>$ 5G)

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** ($>$ 5G)
  - Cooperation (Relaying)

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ “Digital” (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** ($>$ 5G)
  - Cooperation (Relaying) $\rightarrow$ security (?)

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** ($>$ 5G)
  - Cooperation (Relaying) $\rightarrow$ security (?)
  - Het-Nets/Cognitive Radio

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** ($>$ 5G)
  - Cooperation (Relaying) $\rightarrow$ security (?)
  - Het-Nets/Cognitive Radio $\rightarrow$ enough (?)

Past and Future

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** (> 5G)
  - Cooperation (Relaying) $\rightarrow$ security (?)
  - Het-Nets/Cognitive Radio $\rightarrow$ enough (?)
  - Interference Alignment

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** ($>$ 5G)
  - Cooperation (Relaying) $\rightarrow$ security (?)
  - Het-Nets/Cognitive Radio $\rightarrow$ enough (?)
  - Interference Alignment $\rightarrow$ scalability (?)

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** ($>$ 5G)
  - Cooperation (Relaying) $\rightarrow$ security (?)
  - Het-Nets/Cognitive Radio $\rightarrow$ enough (?)
  - Interference Alignment $\rightarrow$ scalability (?)
  - Full Duplex

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** ($>$ 5G)
  - Cooperation (Relaying) $\rightarrow$ security (?)
  - Het-Nets/Cognitive Radio $\rightarrow$ enough (?)
  - Interference Alignment $\rightarrow$ scalability (?)
  - Full Duplex $\rightarrow$ maturity (?)

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ “Digital” (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** ($>$ 5G)
  - Cooperation (Relaying) $\rightarrow$ security (?)
  - Het-Nets/Cognitive Radio $\rightarrow$ enough (?)
  - Interference Alignment $\rightarrow$ scalability (?)
  - Full Duplex $\rightarrow$ maturity (?)
  - Massive MIMO

Past and Future

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** ($> 5G$)
  - Cooperation (Relaying) $\rightarrow$ security (?)
  - Het-Nets/Cognitive Radio $\rightarrow$ enough (?)
  - Interference Alignment $\rightarrow$ scalability (?)
  - Full Duplex $\rightarrow$ maturity (?)
  - Massive MIMO $\rightarrow$ expensive (!)

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** ($>$ 5G)
  - Cooperation (Relaying) $\rightarrow$ security (?)
  - Het-Nets/Cognitive Radio $\rightarrow$ enough (?)
  - Interference Alignment $\rightarrow$ scalability (?)
  - Full Duplex $\rightarrow$ maturity (?)
  - Massive MIMO $\rightarrow$ expensive (!)
- **Bridge of now** (4G $<$?? $<$ 5G)

Past and Future

Bridging Theory and Practice in Cellular Systems

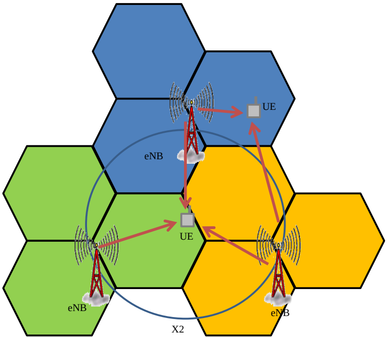
- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** ($>$ 5G)
  - Cooperation (Relaying) $\rightarrow$ security (?)
  - Het-Nets/Cognitive Radio $\rightarrow$ enough (?)
  - Interference Alignment $\rightarrow$ scalability (?)
  - Full Duplex $\rightarrow$ maturity (?)
  - Massive MIMO $\rightarrow$ expensive (!)
- **Bridge of now** (4G $<$?? $<$ 5G)
  - CoMP

Past and Future

Bridging Theory and Practice in Cellular Systems

- **Bridges of old**
  - 2G - Spectrum $\leftarrow$ "Digital" (GSM/CDMA)
  - 3G - More spectrum $\leftarrow$ Spread-spectrum (WCDMA)...
  - 4G - Some more... $\leftarrow$ LTE (OFDMA)
- **Bridges of late** ($>$ 5G)
  - Cooperation (Relaying) $\rightarrow$ security (?)
  - Het-Nets/Cognitive Radio $\rightarrow$ enough (?)
  - Interference Alignment $\rightarrow$ scalability (?)
  - Full Duplex $\rightarrow$ maturity (?)
  - Massive MIMO $\rightarrow$ expensive (!)
- **Bridge of now** (4G $<$?? $<$ 5G)
  - CoMP $\rightarrow$ evolutionary, flexible, mature...

Coordinated Multipoint - CoMP



- Base stations coordinate with each other.
- No receiver cooperation.
- Multiple antennas in transmit and receive side.
- BSs connected via backhaul and CoMP can be perfromed:
  - Joint Processing
  - Coordinated Beamforming
- Network Architecture
  - Centralized Approach
  - Decentralized Approach

Issues with CoMP

- The need of tight synchronization between base stations.
- Signaling overhead on the air interface for the cooperation/coordination of BSs.
- Backhaul speed and latency for the information exchange between BSs
- Limitation in the number of cooperating base stations: Clustering.
- Sensitivity of the channel information feedback from user terminal to the BSs.

Approaching the problem

- We consider a multicell multiuser MIMO systems with coordinating BSs.
- Broader network with conventional size and complexity power.
- Sufficient resources to estimate the channel.
- Consider three different problems
  - Power Minimization Problem Energy Efficiency
  - Max-min SINR Problem Quality of Service
  - Sum Rate Maximization Problem Spectral Efficiency

Power Minimization Problem

Energy Efficiency

$$\mathcal{S}_{\mathsf{P1(MISO)}} = \begin{cases} \underset{\mathbf{U},\mathbf{p}}{\text{minimize}} & \displaystyle\sum_{k=1}^{K} p_k, \\ \text{subject to} & \mathsf{SINR}_k^{\mathsf{DL}} \geq \gamma_k, \quad 1 \leq k \leq K. \end{cases} \tag{1}$$

Max-Min SINR Problem

Quality of Service

$$\mathcal{S}_{\mathsf{P2(MISO)}} = \begin{cases} \underset{\mathbf{U},\mathbf{p}}{\text{maximize}} & \min\limits_{1 \le k \le K} \dfrac{\mathsf{SINR}_k^{\mathsf{DL}}}{\gamma_k} \\ \text{subject to} & \sum\limits_k p_k \le P_{max} \\ & \|\mathbf{u}_k\| = 1, \qquad 1 \le k \le K, \end{cases} \tag{2}$$

Sum-Rate Maximization Problem

Spectral Efficiency

$$\mathcal{S}_{\text{P3-MSE(MISO)}} = \begin{cases} \underset{\mathbf{U},\mathbf{p}}{\text{minimize}} & \sum_{k=1}^{K} w_k \frac{1}{1+\text{SINR}^{\text{DL}}} \\ \text{subject to} & \sum_{k} p_k \leq P_{max} \quad 1 \leq k \leq K. \end{cases} \quad (3)$$

MOTIVATION

Power Minimization Problem

Energy Efficiency

- Example 1: Power minimization problem [Yu&Lan 2007]

$$\begin{array}{ll} \underset{\mathbb{U} \triangleq \{\mathbf{u}_k\}}{\text{minimize}} & \alpha \\ \text{subject to} & \mathbb{E}|x_k|^2 \leq \alpha\, p_k \\ & \text{SINR}_k \geq \gamma_k, \\ \text{given} & \mathbf{h}_k \end{array}$$

$$\text{TX}: \ \mathbf{x}_{N \times 1} = \sum_{k=1}^{K} s_k \mathbf{u}_k \qquad \text{RX}: \ y_k = \mathbf{h}_k \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{|\mathbf{h}_k \cdot \mathbf{u}_k|^2}{\sum_{j \neq k} |\mathbf{h}_k \cdot \mathbf{u}_k|^2 + \sigma^2}$$

Power Minimization Problem

Energy Efficiency

- Example 1: Power minimization problem [Yu&Lan 2007]

$$
\begin{array}{ll}
\underset{\mathbb{U} \triangleq \{\mathbf{u}_k\}}{\text{minimize}} & \alpha \\
\text{subject to} & \mathbb{E}|x_k|^2 \leq \alpha\, p_k \\
 & \text{SINR}_k \geq \gamma_k, \\
\text{given} & \mathbf{h}_k
\end{array}
$$

$$
\text{TX}: \;\; \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} s_k \mathbf{u}_k \qquad \text{RX}: \;\; y_k = \mathbf{h}_k \cdot \mathbf{x} + z_k
$$

$$
\text{SINR}_k = \frac{|\mathbf{h}_k \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} |\mathbf{h}_k \cdot \mathbf{u}_k|^2 + \sigma^2}
$$

- $N = \sum_{b=1}^{B} N_{t_b}$ TX antennas $\rightarrow$ MISO

Power Minimization Problem

Energy Efficiency

- Example 1: Power minimization problem [Yu&Lan 2007]

$$\begin{aligned} \underset{\mathbb{U} \triangleq \{\mathbf{u}_k\}}{\text{minimize}} \quad & \alpha \\ \text{subject to} \quad & \mathbb{E}|x_k|^2 \leq \alpha\, p_k \\ & \text{SINR}_k \geq \gamma_k, \\ \text{given} \quad & \mathbf{h}_k \end{aligned}$$

$$\text{TX}: \; \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} s_k \mathbf{u}_k \qquad \text{RX}: \; y_k = \mathbf{h}_k \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{|\mathbf{h}_k \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} |\mathbf{h}_k \cdot \mathbf{u}_k|^2 + \sigma^2}$$

- $N = \sum_{b=1}^{B} N_{t_b}$ TX antennas $\rightarrow$ MISO
- **Fixed** p_n per-antenna target powers $\rightarrow$ how (?)

Power Minimization Problem

Energy Efficiency

- Example 1: Power minimization problem [Yu&Lan 2007]

$$\begin{aligned} \underset{\mathbb{U} \triangleq \{\mathbf{u}_k\}}{\text{minimize}} \quad & \alpha \\ \text{subject to} \quad & \mathbb{E}|x_k|^2 \leq \alpha\, p_k \\ & \text{SINR}_k \geq \gamma_k, \\ \text{given} \quad & \mathbf{h}_k \end{aligned}$$

$$\text{TX}: \; \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} s_k \mathbf{u}_k \qquad \text{RX}: \; y_k = \mathbf{h}_k \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{|\mathbf{h}_k \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} |\mathbf{h}_k \cdot \mathbf{u}_k|^2 + \sigma^2}$$

- $N = \sum_{b=1}^{B} N_{t_b}$ TX antennas $\rightarrow$ MISO
- **Fixed** p_n per-antenna target powers $\rightarrow$ how (?)
- **Per user** γ_k target SINRs $\rightarrow$ QoS balancing (?)

Power Minimization Problem

- Example 1: Power minimization problem [Yu&Lan 2007]

$$\begin{aligned} \underset{\mathbb{U} \triangleq \{\mathbf{u}_k\}}{\text{minimize}} \quad & \alpha \\ \text{subject to} \quad & \mathbb{E}|x_k|^2 \leq \alpha\, p_k \\ & \text{SINR}_k \geq \gamma_k, \\ \text{given} \quad & \mathbf{h}_k \end{aligned}$$

$$\text{TX}: \ \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} s_k \mathbf{u}_k \qquad \text{RX}: \ y_k = \mathbf{h}_k \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{|\mathbf{h}_k \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} |\mathbf{h}_k \cdot \mathbf{u}_k|^2 + \sigma^2}$$

  - $N = \sum_{b=1}^{B} N_{t_b}$ TX antennas $\rightarrow$ MISO
  - **Fixed** p_n per-antenna target powers $\rightarrow$ how (?)
  - **Per user** γ_k target SINRs $\rightarrow$ QoS balancing (?)
  - **Perfectly known** $\mathbf{h}_k$ for all users $\rightarrow$ overhead (!)

Power Minimization Problem

Energy Efficiency

- Example 2: Power minimization problem [Song et al. 2007]

$$\begin{aligned} \underset{\mathbf{p}>0,\mathbb{V},\mathbb{U}}{\text{minimize}} \quad & \sum_{k=1}^{K} w_k \, p_k \\ \text{subject to} \quad & \text{SINR}_k \geq \gamma_k \\ \text{given} \quad & \mathbf{H}_{kj} \text{ and } w_k \end{aligned}$$

$$\text{TX}: \ \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \text{RX}: \ y_k = \mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{p_k |\mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{v}_k^H \cdot \mathbf{H}_{kj} \cdot \mathbf{u}_k|^2 + \sigma^2}$$

Power Minimization Problem

- Example 2: Power minimization problem [Song et al. 2007]

$$\begin{array}{ll} \underset{\mathbf{p}>0,\mathbb{V},\mathbb{U}}{\text{minimize}} & \sum_{k=1}^{K} w_k \, p_k \\ \text{subject to} & \text{SINR}_k \geq \gamma_k \\ \text{given} & \mathbf{H}_{kj} \text{ and } w_k \end{array}$$

$$\text{TX}: \; \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \text{RX}: \; y_k = \mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{p_k |\mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{v}_k^H \cdot \mathbf{H}_{kj} \cdot \mathbf{u}_k|^2 + \sigma^2}$$

- ~~$N = \sum_{b=1}^{B} N_{t_b}$ TX antennas~~ $\rightarrow$ MIMO

Power Minimization Problem

Energy Efficiency

- Example 2: Power minimization problem [Song et al. 2007]

$$\begin{array}{ll} \underset{\mathbf{p}>0,\mathbb{V},\mathbb{U}}{\text{minimize}} & \sum_{k=1}^{K} w_k \, p_k \\ \text{subject to} & \text{SINR}_k \geq \gamma_k \\ \text{given} & \mathbf{H}_{kj} \text{ and } w_k \end{array}$$

$$\text{TX}: \; \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \text{RX}: \; y_k = \mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{p_k |\mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{v}_k^H \cdot \mathbf{H}_{kj} \cdot \mathbf{u}_k|^2 + \sigma^2}$$

- ~~$N = \sum_{b=1}^{B} N_{t_b}$ TX antennas~~ $\rightarrow$ MIMO
- ~~**Fixed** p_n per-antenna target powers~~ $\rightarrow$ optimized **per user** p_k
- **Known** weight per user w_k $\rightarrow$ how (?)

Power Minimization Problem

Energy Efficiency

- Example 2: Power minimization problem [Song et al. 2007]

$$\begin{aligned} \underset{\mathbf{p}>0,\mathbb{V},\mathbb{U}}{\text{minimize}} \quad & \sum_{k=1}^{K} w_k \, p_k \\ \text{subject to} \quad & \text{SINR}_k \geq \gamma_k \\ \text{given} \quad & \mathbf{H}_{kj} \text{ and } w_k \end{aligned}$$

$$\text{TX}: \ \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \text{RX}: \ y_k = \mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{p_k |\mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{v}_k^H \cdot \mathbf{H}_{kj} \cdot \mathbf{u}_k|^2 + \sigma^2}$$

- ~~$N = \sum_{b=1}^{B} N_{t_b}$ TX antennas~~ $\rightarrow$ MIMO
- ~~**Fixed** p_n per-antenna target powers~~ $\rightarrow$ optimized **per user** p_k
- **Known** weight per user w_k $\rightarrow$ how (?)
- **Per user** γ_k target SINRs $\rightarrow$ QoS balancing (?)

Power Minimization Problem

Energy Efficiency

- Example 2: Power minimization problem [Song et al. 2007]

$$\begin{aligned} \underset{\mathbf{p}>0,\mathbb{V},\mathbb{U}}{\text{minimize}} \quad & \sum_{k=1}^{K} w_k\, p_k \\ \text{subject to} \quad & \text{SINR}_k \geq \gamma_k \\ \text{given} \quad & \mathbf{H}_{kj} \text{ and } w_k \end{aligned}$$

$$\text{TX}: \ \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \text{RX}: \ y_k = \mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{p_k |\mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{v}_k^H \cdot \mathbf{H}_{kj} \cdot \mathbf{u}_k|^2 + \sigma^2}$$

- ~~$N = \sum_{b=1}^{B} N_{t_b}$ TX antennas~~ $\rightarrow$ MIMO
- ~~**Fixed** p_n per-antenna target powers~~ $\rightarrow$ optimized **per user** p_k
- **Known** weight per user w_k $\rightarrow$ how (?)
- **Per user** γ_k target SINRs $\rightarrow$ QoS balancing (?)
- **Perfectly known** $\mathbf{H}_{kj}$ for all users $\rightarrow$ overhead (!)

Max-min SINR Problem

Quality of Service

- Example 3: Min-max SINR problem [Huang et al. 2011]

$$\begin{array}{ll} \underset{\mathbf{p}>0,\mathbb{U}}{\text{maximize}} & \min_{\forall k} \ \text{SINR}_k \\ \text{subject to} & \|\mathbf{p}\| \leq P \\ & \|\mathbf{u}_k\| = 1 \\ \text{given} & \mathbf{h}_k \end{array}$$

$$\text{TX}: \ \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \text{RX}: \ y_k = \mathbf{h}_k \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{p_k |\mathbf{h}_k \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{h}_j \cdot \mathbf{u}_k|^2 + \sigma^2}$$

Max-min SINR Problem

Quality of Service

- Example 3: Min-max SINR problem [Huang et al. 2011]

$$\begin{array}{ll} \underset{\mathbf{p}>0,\mathbb{U}}{\text{maximize}} & \min_{\forall k} \ \text{SINR}_k \\ \text{subject to} & \|\mathbf{p}\| \leq P \\ & \|\mathbf{u}_k\| = 1 \\ \text{given} & \mathbf{h}_k \end{array}$$

$$\text{TX}: \ \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \text{RX}: \ y_k = \mathbf{h}_k \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{p_k |\mathbf{h}_k \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{h}_j \cdot \mathbf{u}_k|^2 + \sigma^2}$$

- $N = \sum_{b=1}^{B} N_{t_b}$ TX antennas $\rightarrow$ MISO

Max-min SINR Problem

Quality of Service

- Example 3: Min-max SINR problem [Huang et al. 2011]

$$\begin{array}{ll} \underset{\mathbf{p}>0,\mathbb{U}}{\text{maximize}} & \min_{\forall k} \ \text{SINR}_k \\ \text{subject to} & \|\mathbf{p}\| \le P \\ & \|\mathbf{u}_k\| = 1 \\ \text{given} & \mathbf{h}_k \end{array}$$

$$\text{TX}: \ \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \text{RX}: \ y_k = \mathbf{h}_k \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{p_k |\mathbf{h}_k \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{h}_j \cdot \mathbf{u}_k|^2 + \sigma^2}$$

  - $N = \sum_{b=1}^{B} N_{t_b}$ TX antennas → MISO
  - ~~**Fixed** p_n per-antenna target powers~~ → optimized **per user** p_k

Max-min SINR Problem

Quality of Service

- Example 3: Min-max SINR problem [Huang et al. 2011]

$$\begin{aligned} \underset{\mathbf{p}>0,\mathbb{U}}{\text{maximize}} \quad & \min_{\forall k} \ \mathsf{SINR}_k \\ \text{subject to} \quad & \|\mathbf{p}\| \leq P \\ & \|\mathbf{u}_k\| = 1 \\ \text{given} \quad & \mathbf{h}_k \end{aligned}$$

$$\mathsf{TX}: \ \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \mathsf{RX}: \ y_k = \mathbf{h}_k \cdot \mathbf{x} + z_k$$

$$\mathsf{SINR}_k = \frac{p_k |\mathbf{h}_k \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{h}_j \cdot \mathbf{u}_k|^2 + \sigma^2}$$

  - $N = \sum_{b=1}^{B} N_{t_b}$ TX antennas → MISO
  - ~~**Fixed** p_n per-antenna target powers~~ → optimized **per user** p_k
  - ~~**Per user** γ_k target SINRs~~ → minimum QoS

Max-min SINR Problem

Quality of Service

- Example 3: Min-max SINR problem [Huang et al. 2011]

$$\begin{aligned} \underset{\mathbf{p}>0,\mathbb{U}}{\text{maximize}} \quad & \min_{\forall k} \ \text{SINR}_k \\ \text{subject to} \quad & \|\mathbf{p}\| \leq P \\ & \|\mathbf{u}_k\| = 1 \\ \text{given} \quad & \mathbf{h}_k \end{aligned}$$

$$\text{TX}: \ \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \text{RX}: \ y_k = \mathbf{h}_k \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{p_k |\mathbf{h}_k \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{h}_j \cdot \mathbf{u}_k|^2 + \sigma^2}$$

  - $N = \sum_{b=1}^{B} N_{t_b}$ TX antennas → MISO
  - ~~**Fixed** p_n per-antenna target powers~~ → optimized **per user** p_k
  - ~~**Per user** γ_k target SINRs~~ → minimum QoS
  - **Perfectly known** $\mathbf{h}_k$ for all → overhead (!)

Max-min SINR Problem

Quality of Service

- Example 4: Min-max SINR problem [Cai et al. 2011]

$$\begin{array}{ll} \underset{\mathbf{p}>0,\mathbb{U},\mathbb{V}}{\text{maximize}} & \min_{\forall k} \dfrac{\text{SINR}_k}{\alpha_k} \\ \text{subject to} & \mathbf{w}_\ell \cdot \mathbf{p} \le P_\ell,\ \ell \in \mathcal{L} \,\Big|\, |\mathcal{L}| < K \\ \text{given} & \mathbf{H}_{kj},\ \mathbf{w}_k \text{ and } \boldsymbol{\alpha} \end{array}$$

$$\text{TX}: \ \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \text{RX}: \ y_k = \mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{p_k |\mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{v}_k^H \cdot \mathbf{H}_{kj} \cdot \mathbf{u}_k|^2 + \sigma^2}$$

Max-min SINR Problem

Quality of Service

- Example 4: Min-max SINR problem [Cai et al. 2011]

$$\begin{array}{ll} \underset{\mathbf{p}>0,\mathbb{U},\mathbb{V}}{\text{maximize}} & \min_{\forall k} \frac{\mathsf{SINR}_k}{\alpha_k} \\ \text{subject to} & \mathbf{w}_\ell \cdot \mathbf{p} \leq P_\ell,\ \ell \in \mathcal{L} \,\Big|\, |\mathcal{L}| < K \\ \text{given} & \mathbf{H}_{kj},\ \mathbf{w}_k \text{ and } \boldsymbol{\alpha} \end{array}$$

$$\mathsf{TX}: \ \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \mathsf{RX}: \ y_k = \mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{x} + z_k$$

$$\mathsf{SINR}_k = \frac{p_k |\mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{v}_k^H \cdot \mathbf{H}_{kj} \cdot \mathbf{u}_k|^2 + \sigma^2}$$

  - ~~$N = \sum_{b=1}^{B} N_{t_b}$ TX antennas~~ $\rightarrow$ MIMO

Max-min SINR Problem

Quality of Service

- Example 4: Min-max SINR problem [Cai et al. 2011]

$$\begin{aligned} \underset{\mathbf{p}>0,\mathbb{U},\mathbb{V}}{\text{maximize}} \quad & \min_{\forall k} \frac{\text{SINR}_k}{\alpha_k} \\ \text{subject to} \quad & \mathbf{w}_\ell \cdot \mathbf{p} \le P_\ell,\ \ell \in \mathcal{L} \,\Big|\, |\mathcal{L}| < K \\ \text{given} \quad & \mathbf{H}_{kj},\ \mathbf{w}_k \text{ and } \boldsymbol{\alpha} \end{aligned}$$

$$\text{TX}: \ \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \text{RX}: \ y_k = \mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{p_k |\mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{v}_k^H \cdot \mathbf{H}_{kj} \cdot \mathbf{u}_k|^2 + \sigma^2}$$

  - ~~$N = \sum_{b=1}^{B} N_{t_b}$ TX antennas~~ $\rightarrow$ MIMO
  - ~~**Fixed** p_n per-antenna target powers~~ $\rightarrow$ optimized **per user** p_k
  - **Known** weight vectors per user $\mathbf{w}_k$ and scores α_k $\rightarrow$ how (?)

Max-min SINR Problem

Quality of Service

- Example 4: Min-max SINR problem [Cai et al. 2011]

$$\underset{\mathbf{p}>0,\mathbb{U},\mathbb{V}}{\text{maximize}} \quad \min_{\forall k} \frac{\text{SINR}_k}{\alpha_k}$$

$$\text{subject to} \quad \mathbf{w}_\ell \cdot \mathbf{p} \le P_\ell,\ \ell \in \mathcal{L} \,\Big|\, |\mathcal{L}| < K$$

$$\text{given} \quad \mathbf{H}_{kj},\ \mathbf{w}_k \text{ and } \boldsymbol{\alpha}$$

$$\text{TX}: \ \mathbf{x}_{N\times 1} = \sum_{k=1}^{K} \sqrt{p_k}\, s_k \mathbf{u}_k \qquad \text{RX}: \ y_k = \mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{x} + z_k$$

$$\text{SINR}_k = \frac{p_k |\mathbf{v}_k^H \cdot \mathbf{H}_{kk} \cdot \mathbf{u}_k|^2}{\sum_{j\neq k} p_j |\mathbf{v}_k^H \cdot \mathbf{H}_{kj} \cdot \mathbf{u}_k|^2 + \sigma^2}$$

  - ~~$N = \sum_{b=1}^{B} N_{t_b}$ TX antennas~~ → MIMO
  - ~~**Fixed** p_n per-antenna target powers~~ → optimized **per user** p_k
  - **Known** weight vectors per user $\mathbf{w}_k$ and scores α_k → how (?)
  - **Perfectly known** $\mathbf{H}_{kj}$ for all users → overhead (!)

Sum-Rate Maximization Problem

Spectral Efficiency

- Example 5: Sum-rate maximization problem [Tran et al. 2012]

$$
\begin{aligned}
\underset{\mathbf{t},\mathbb{U}}{\text{maximize}} \quad & \prod_{k=1}^{K} t_k \\
\text{subject to} \quad & \mathsf{SINR}_k \geq t_k^{1/\alpha_k} - 1 \\
& \sum_{k=1}^{K} \|\mathbf{u}_k\|^2 \leq P \\
\text{given} \quad & \mathbf{h}_k, P \text{ and } \boldsymbol{\alpha}
\end{aligned}
$$

$$\alpha_k \log_2(1 + \mathsf{SINR}_k) \longrightarrow (1 + \mathsf{SINR}_k)^{\alpha_k} \longrightarrow t_k$$

Sum-Rate Maximization Problem

Spectral Efficiency

- Example 5: Sum-rate maximization problem [Tran et al. 2012]

$$
\begin{aligned}
\underset{\mathbf{t},\mathbb{U}}{\text{maximize}} \quad & \prod_{k=1}^{K} t_k \\
\text{subject to} \quad & \mathsf{SINR}_k \geq t_k^{1/\alpha_k} - 1 \\
& \sum_{k=1}^{K} \|\mathbf{u}_k\|^2 \leq P \\
\text{given} \quad & \mathbf{h}_k, P \text{ and } \boldsymbol{\alpha}
\end{aligned}
$$

$$\alpha_k \log_2(1 + \mathsf{SINR}_k) \longrightarrow (1 + \mathsf{SINR}_k)^{\alpha_k} \longrightarrow t_k$$

- $N = \sum_{b=1}^{B} N_{t_b}$ TX antennas → MISO

Sum-Rate Maximization Problem

Spectral Efficiency

- Example 5: Sum-rate maximization problem [Tran et al. 2012]

$$\begin{aligned}
\underset{\mathbf{t},\mathbb{U}}{\text{maximize}} \quad & \prod_{k=1}^{K} t_k \\
\text{subject to} \quad & \mathsf{SINR}_k \geq t_k^{1/\alpha_k} - 1 \\
& \sum_{k=1}^{K} \|\mathbf{u}_k\|^2 \leq P \\
\text{given} \quad & \mathbf{h}_k, P \text{ and } \boldsymbol{\alpha}
\end{aligned}$$

$$\alpha_k \log_2(1 + \mathsf{SINR}_k) \longrightarrow (1 + \mathsf{SINR}_k)^{\alpha_k} \longrightarrow t_k$$

- $N = \sum_{b=1}^{B} N_{t_b}$ TX antennas → MISO
- ~~**Fixed** p_n per-antenna target powers~~ → optimized **total** p_k
- **Known** scores α_k → how (?)

Sum-Rate Maximization Problem

Spectral Efficiency

- Example 5: Sum-rate maximization problem [Tran et al. 2012]

$$
\begin{aligned}
\underset{\mathbf{t},\mathbb{U}}{\text{maximize}} \quad & \prod_{k=1}^{K} t_k \\
\text{subject to} \quad & \mathsf{SINR}_k \geq t_k^{1/\alpha_k} - 1 \\
& \sum_{k=1}^{K} \|\mathbf{u}_k\|^2 \leq P \\
\text{given} \quad & \mathbf{h}_k, P \text{ and } \boldsymbol{\alpha}
\end{aligned}
$$

$$\alpha_k \log_2(1 + \mathsf{SINR}_k) \longrightarrow (1 + \mathsf{SINR}_k)^{\alpha_k} \longrightarrow t_k$$

- $N = \sum_{b=1}^{B} N_{t_b}$ TX antennas $\rightarrow$ MISO
- ~~**Fixed** p_n per-antenna target powers~~ $\rightarrow$ optimized **total** p_k
- **Known** scores α_k $\rightarrow$ how (?)
- **Perfectly known** $\mathbf{h}_k$ for all users $\rightarrow$ overhead (!)

Sum-Rate Maximization Problem

Spectral Efficiency

- Example 6: Sum-rate maximization problem [Park et al. 2013]

$$\begin{aligned} \underset{\mathbb{V}\triangleq\{\mathbf{V}_k\}}{\text{maximize}} \quad & \sum_{k=1}^{K} w_k \log_2 |\mathbf{I}_{N_r} + (\sigma_k^2\mathbf{I} + \mathbf{\Phi}_k)^{-1}\mathbf{H}_{kk}\mathbf{V}_k\mathbf{V}_k\mathbf{H}_{kk}^H| \\ \text{subject to} \quad & \|\mathbf{H}_{jk}\mathbf{V}_k\|^2 \le \alpha_{jk}\sigma_j^2 \\ & \|\mathbf{V}_k\|^2 \le p_k \\ \text{given} \quad & \mathbf{H}_{jk}, \mathbf{w}, \mathbf{p} \text{ and } \boldsymbol{\alpha} \end{aligned}$$

$$\mathbf{\Phi}_k = \sum_{k=1}^{K} \mathbf{H}_{kj}\mathbf{V}_j\mathbf{V}_j^H\mathbf{H}_{kj}^H$$

Sum-Rate Maximization Problem

Spectral Efficiency

- Example 6: Sum-rate maximization problem [Park et al. 2013]

$$\begin{array}{ll} \underset{\mathbb{V} \triangleq \{\mathbf{V}_k\}}{\text{maximize}} & \sum_{k=1}^{K} w_k \log_2 |\mathbf{I}_{N_r} + (\sigma_k^2 \mathbf{I} + \mathbf{\Phi}_k)^{-1} \mathbf{H}_{kk} \mathbf{V}_k \mathbf{V}_k \mathbf{H}_{kk}^H| \\ \text{subject to} & \|\mathbf{H}_{jk} \mathbf{V}_k\|^2 \leq \alpha_{jk} \sigma_j^2 \\ & \|\mathbf{V}_k\|^2 \leq p_k \\ \text{given} & \mathbf{H}_{jk}, \mathbf{w}, \mathbf{p} \text{ and } \boldsymbol{\alpha} \end{array}$$

$$\mathbf{\Phi}_k = \sum_{k=1}^{K} \mathbf{H}_{kj} \mathbf{V}_j \mathbf{V}_j^H \mathbf{H}_{kj}^H$$

  - ~~$N = \sum_{b=1}^{B} N_{t_b}$ TX antennas~~ $\rightarrow$ MIMO

Sum-Rate Maximization Problem

Spectral Efficiency

- Example 6: Sum-rate maximization problem [Park et al. 2013]

$$
\begin{aligned}
\underset{\mathbb{V}\triangleq\{\mathbf{V}_k\}}{\text{maximize}} \quad & \sum_{k=1}^{K} w_k \log_2 \left|\mathbf{I}_{N_r} + (\sigma_k^2\mathbf{I} + \mathbf{\Phi}_k)^{-1}\mathbf{H}_{kk}\mathbf{V}_k\mathbf{V}_k\mathbf{H}_{kk}^H\right| \\
\text{subject to} \quad & \|\mathbf{H}_{jk}\mathbf{V}_k\|^2 \leq \alpha_{jk}\sigma_j^2 \\
& \|\mathbf{V}_k\|^2 \leq p_k \\
\text{given} \quad & \mathbf{H}_{jk}, \mathbf{w}, \mathbf{p} \text{ and } \boldsymbol{\alpha}
\end{aligned}
$$

$$
\mathbf{\Phi}_k = \sum_{k=1}^{K} \mathbf{H}_{kj}\mathbf{V}_j\mathbf{V}_j^H\mathbf{H}_{kj}^H
$$

  - ~~$N = \sum_{b=1}^{B} N_{t_b}$ TX antennas~~ $\rightarrow$ MIMO
  - ~~**Fixed** p_n per-antenna target powers~~ $\rightarrow$ optimized **per user** p_k
  - **Known** weights $\mathbf{w}$, target powers p_k and scores α_k $\rightarrow$ how (?)

Sum-Rate Maximization Problem

Spectral Efficiency

- Example 6: Sum-rate maximization problem [Park et al. 2013]

$$\begin{array}{ll}
\underset{\mathbb{V}\triangleq\{\mathbf{V}_k\}}{\text{maximize}} & \sum_{k=1}^{K} w_k \log_2 \left|\mathbf{I}_{N_r} + (\sigma_k^2 \mathbf{I} + \mathbf{\Phi}_k)^{-1} \mathbf{H}_{kk} \mathbf{V}_k \mathbf{V}_k \mathbf{H}_{kk}^H\right| \\
\text{subject to} & \|\mathbf{H}_{jk}\mathbf{V}_k\|^2 \leq \alpha_{jk}\sigma_j^2 \\
 & \|\mathbf{V}_k\|^2 \leq p_k \\
\text{given} & \mathbf{H}_{jk}, \mathbf{w}, \mathbf{p} \text{ and } \boldsymbol{\alpha}
\end{array}$$

$$\mathbf{\Phi}_k = \sum_{k=1}^{K} \mathbf{H}_{kj} \mathbf{V}_j \mathbf{V}_j^H \mathbf{H}_{kj}^H$$

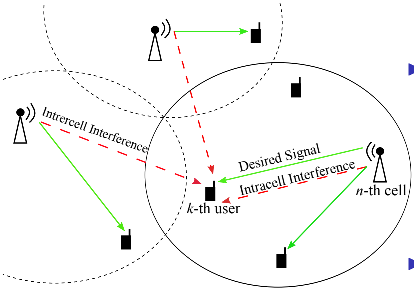
- ~~$N = \sum_{b=1}^{B} N_{t_b}$ TX antennas~~ $\rightarrow$ MIMO
- ~~**Fixed** p_n per-antenna target powers~~ $\rightarrow$ optimized **per user** p_k
- **Known** weights $\mathbf{w}$, target powers p_k and scores α_k $\rightarrow$ how (?)
- **Perfectly known** $\mathbf{H}_{kj}$ for all users $\rightarrow$ overhead (!)

Comprehensive Review

		Instantaneous Perfect CSIT	Instantaneous Imperfect CSIT	Statistical CSIT	Covariance Information
P1	MISO	[20-25, 62]	[26]	[27-30]	[31-33,63]
	MIMO	[22,38,64]			
P2	MISO	[34-37,42]	[43]	[42-44-47]	
	MIMO	[38-41]			
P3	MISO	[48,49,51-55,57,65,66]	[60]	[61]	
	MIMO	[57-59,67-69]			

SOME TOOLS

System Model



- The transmitted signal

$$\mathbf{x} = \sum_{j=1}^{K} \sqrt{p_j}\mathbf{u}_j s_j$$

- The received signal

$$y_k = \mathbf{h}_k^{\mathsf{H}} \left(\sum_{j=1}^{K} \sqrt{p_j}\mathbf{u}_j s_j \right) + z_k$$

- The SINR of user k

$$\Phi_k^{\mathsf{DL}} = \frac{p_k |\mathbf{h}_k^{\mathsf{H}} \mathbf{u}_k|^2}{\sum_{j=1, j\neq k}^{K} p_j |\mathbf{h}_k^{\mathsf{H}} \mathbf{u}_j|^2 + \sigma_{z_k}^2}.$$

Perron-Frobenius Theorem

- Characterizes eigenvectors and eigenvalues of non-negative matrices.
- Possible solution for power minimization in wireless network.
- SINR Constraint

$$\frac{p_k|\mathbf{h}_k^{\mathsf{H}}\mathbf{u}_k|^2}{\sum_{j=1,j\neq k}^{K} p_j|\mathbf{h}_k^{\mathsf{H}}\mathbf{u}_j|^2 + \sigma_{z_k}^2} \geq \gamma_k. \quad (4)$$

- The SINR constraint can be re-written as

$$p_k G_{kk} \geq \gamma_k \left(\sum_{j=1,j\neq k}^{K} p_j G_{kj} + \sigma_{z_k}^2 \right), \quad (5)$$

where $G_{jk} = |\mathbf{h}_j \mathbf{u}_k|^2$.

Perron-Frobenius Theorem

- We now define two matrices $\mathbf{D}$ and $\mathbf{G}$

$$\mathbf{D} = \begin{bmatrix} \frac{\gamma_1}{G_{11}} & 0 & \cdots & 0 \\ 0 & \frac{\gamma_2}{G_{22}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\gamma_K}{G_{KK}} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} 0 & G_{12} & \cdots & G_{1K} \\ G_{21} & 0 & \cdots & G_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ G_{K1} & G_{K2} & \cdots & 0 \end{bmatrix},$$

- The SINR constraint in the matrix form can be written as

$$(\mathbf{I} - \mathbf{DG})\mathbf{p} \geq \mathbf{Dn},$$

where $\mathbf{p} = [p_1, p_2, \cdots, p_K]^\mathsf{T}$, and $\mathbf{n} = [\sigma_{z_1}^2, \sigma_{z_2}^2, \cdots, \sigma_{z_k}^2]^\mathsf{T}$.

- The optimal solution

$$\mathbf{p}^* = (\mathbf{I} - \mathbf{DG})^{-1}\mathbf{Dn}$$

Perron-Frobenius Theorem

- The necessary and sufficient conditions for $\mathbf{p}$ to be positive

$$(\mathbf{I} - \underbrace{\mathbf{DG}}_{\mathbf{A}})^{-1} \geq 0 \quad \text{iff} \quad \rho(\mathbf{A}) = |\lambda_{max}(\mathbf{A})| < 1.$$

Conic Programming

- Convex optimization tools are widely used in communication / signal processing algorithms.
- Formulation of the non-convex problems to convex problems.
- Efficient use of convex optimization tools as CVX and SeDuMi.
- Consider the SINR constraint

$$p_k|\mathbf{h}_k^{\mathsf{H}}\mathbf{u}_k|^2 - \gamma_k \sum_{j=1, j\neq k}^{K} p_j|\mathbf{h}_k^{\mathsf{H}}\mathbf{u}_j|^2 \geq \gamma_k \left(\sigma_{z_k}^2\right). \qquad (6)$$

- The constraint is a quadratic optimization problem with quadratic non-convex constraints.

Beamforming Problem as SDP

Consider a power minimization problem

$$\mathcal{S}_{\text{P1(MISO)}} = \begin{cases} \underset{\mathbf{U}}{\text{minimize}} & \sum_{k=1}^{K} \|\mathbf{u}_k\|^2, \\ \text{subject to} & \dfrac{|\mathbf{h}_k^{\mathsf{H}}\mathbf{u}_k|^2}{\sum_{j=1, j\neq k}^{K} |\mathbf{h}_k^{\mathsf{H}}\mathbf{u}_j|^2 + \sigma_{z_k}^2} \geq \gamma_k, \quad 1 \leq k \leq K. \end{cases} \tag{7}$$

Let us define $\mathbf{A}_k = \mathbf{u}_k\mathbf{u}_k^{\mathsf{H}}$ and $\mathbf{R}_k = \mathbf{h}_k\mathbf{h}_k^{\mathsf{H}}$. The optimization problem now can be transformed as

$$\mathcal{S}_{\text{P1-SDP(MISO)}} = \begin{cases} \underset{\{\mathbf{A}_k\}_{k=1}^{K}}{\text{minimize}} & \sum_{k=1}^{K} \text{Trace}(\mathbf{A}_k), \\ \text{subject to} & \text{Trace}(\mathbf{R}_k\mathbf{A}_k) - \gamma_k \sum_{j\neq k}^{K} \text{Trace}(\mathbf{R}_k\mathbf{A}_j) \geq \gamma_k\sigma_{z_k}^2 \\ & \mathbf{A}_k \succeq 0, \quad \text{rank}(\mathbf{A}_k) = 1, \quad 1 \leq k \leq K. \end{cases} \tag{8}$$

Beamforming Problem as SOCP

- An arbitrary phase rotation does not affect the SINR of the user.
- Considering only the real part of the gain matrix, the SINR constraint can be re-written as

$$\left(1+\frac{1}{\gamma_k}\right)|\mathbf{h}_k^{\mathsf{H}}\mathbf{u}_k|^2 \geq \left\| \begin{array}{c} \mathbf{h}_i^{\mathsf{H}}\mathbf{U} \\ \sigma_{z_k}^2 \end{array} \right\|^2, \quad 1 \leq k \leq K. \tag{9}$$

- And, the optimization problem thus becomes

$$\mathcal{S}_{\text{P1-SOCP(MISO)}} = \begin{cases} \text{minimize} & \tau \\ \text{subject to} & \sqrt{\left(1+\frac{1}{\gamma_k}\right)}\mathbf{h}_k^{\mathsf{H}}\mathbf{u}_k \geq \left\| \begin{array}{c} \mathbf{h}_i^{\mathsf{H}}\mathbf{U} \\ \sigma_{z_k}^2 \end{array} \right\| \\ & \sum_{k=1}^{K} \|\mathbf{u}_k\| \leq \tau, \quad 1 \leq k \leq K, \end{cases} \tag{10}$$

UL-DL duality via Lagrangian Duality

- The Lagrangian of the SINR constraint can be written as

$$L(\mathbf{u}_k, \lambda_k) = \sum_{k=1}^{K} \lambda_k \sigma_{z_k}^2 + \sum_{k=1}^{K} \mathbf{u}_k^{\mathsf{H}} \Big(\mathbf{I} + \sum_{\substack{j=1 \\ j \neq k}}^{K} \lambda_j \mathbf{h}_j \mathbf{h}_j^{\mathsf{H}} - \frac{\lambda_k}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^{\mathsf{H}} \Big) \mathbf{u}_k \quad (11)$$

- The dual objective is

$$g(\lambda_k) = \min_{\mathbf{u}_k} \quad L\left(\mathbf{u}_k, \lambda_k\right) \quad (12)$$

- The Lagrangian dual problem is

$$\begin{aligned} \text{maximize} \quad & \sum_{k=1}^{K} \lambda_k \sigma_{z_k}^2 \\ \text{subject to} \quad & \sum_{j=1}^{K} \lambda_j \mathbf{h}_j \mathbf{h}_j^{\mathsf{H}} + \mathbf{I} \succeq \left(1 + \frac{1}{\gamma_k}\right) \lambda_k \mathbf{h}_k \mathbf{h}_k^{\mathsf{H}} \end{aligned} \quad (13)$$

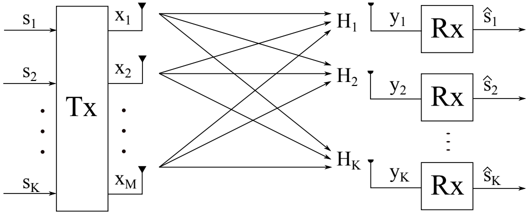
UL-DL duality via Lagrangian Duality

- The sum power minimization problem for uplink

$$\begin{aligned} \text{minimize} \quad & \sum_{k=1}^{K} \rho_k \\ \text{subject to} \quad & \sum_{j=1}^{K} \rho_j \mathbf{h}_j \mathbf{h}_j^{\mathsf{H}} + \sigma_{z_k}^2 \mathbf{I} \preceq \left(1 + \frac{1}{\gamma_k}\right) \rho_k \mathbf{h}_k \mathbf{h}_k^{\mathsf{H}} \end{aligned} \tag{14}$$

- For $\rho_k = \lambda_k \sigma_{z_k}^2$, Eq (13) and Eq (14) are identical.
- Eq (13) and Eq (14) gives the same solution.
- The dual variables of the downlink problem have the interpretation of being the uplink power scaled by the noise variance.

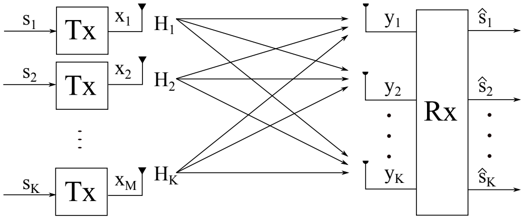
Duality Theory for UL-DL Beamforming



- The downlink SINR is given as

$$\hat{\Phi}_k^{\mathsf{DL}} = \frac{p_k |\hat{\mathbf{h}}_k^{\mathsf{H}} \mathbf{u}_k|^2}{\sum_{j=1, j \neq k}^{K} p_j |\hat{\mathbf{h}}_k^{\mathsf{H}} \mathbf{u}_j|^2 + \sigma_{z_i}^2}. \tag{15}$$

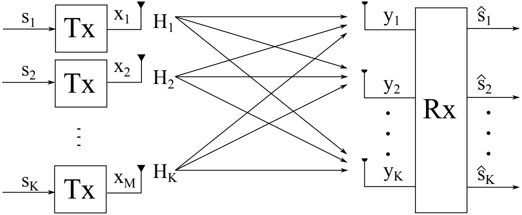
Duality Theory for UL-DL Beamforming



- The uplink SINR is given as

$$\hat{\Phi}_k^{\mathsf{UL}} = \frac{q_k |\hat{\mathbf{h}}_k^{\mathsf{H}} \mathbf{u}_k|^2}{\mathbf{u}_k^H \left(\sum_{j=1, j \neq k}^{K} q_j \hat{\mathbf{h}}_j \hat{\mathbf{h}}_j^{\mathsf{H}} + \sigma_{z_k}^2 \mathbf{I} \right) \mathbf{u}_k}. \quad (16)$$

Duality Theory for UL-DL Beamforming



- The uplink SINR is given as

$$\hat{\Phi}_k^{\mathsf{UL}} = \frac{q_k |\hat{\mathbf{h}}_k^{\mathsf{H}} \mathbf{u}_k|^2}{\mathbf{u}_k^H \left(\sum_{j=1, j \neq k}^{K} q_j \hat{\mathbf{h}}_j \hat{\mathbf{h}}_j^{\mathsf{H}} + \sigma_{z_k}^2 \mathbf{I} \right) \mathbf{u}_k}. \tag{16}$$

Uplink SINRs are only coupled by transmission powers, however, the downlink SINRs are additionally coupled by beamforming vectors, making direct optimization difficult.

Duality-Downlink Power Assignment

- For a fixed beamformers, power optimization reduces to

$$\mathcal{S}^{DL}(\tilde{\mathbf{U}}, P_{max}) = \begin{cases} \underset{\mathbf{p}}{\text{maximize}} & \min_{1 \leq k \leq K} \frac{\text{SINR}_k^{(DL)}}{\gamma_k} \\ \text{subject to} & \sum_k p_k = P_{max} \end{cases} \tag{17}$$

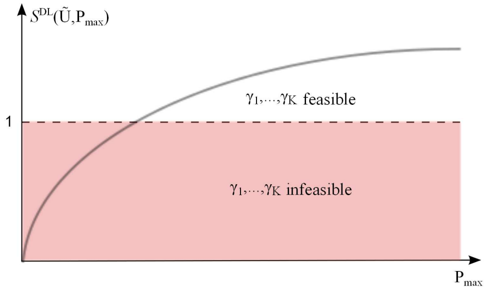
- $\mathcal{S}^{DL}(\mathbf{U}, P_{max})$ is strictly monotonically increasing in P_{max}.

Duality-Downlink Power Assignment

- For a fixed beamformers, power optimization reduces to

$$\mathcal{S}^{DL}(\tilde{\mathbf{U}}, P_{max}) = \begin{cases} \underset{\mathbf{p}}{\text{maximize}} & \min_{1\leq k\leq K} \frac{\text{SINR}_k^{(DL)}}{\gamma_k} \\ \text{subject to} & \sum_k p_k = P_{max} \end{cases} \quad (17)$$

- $\mathcal{S}^{DL}(\mathbf{U}, P_{max})$ is strictly monotonically increasing in P_{max}.



Duality-Downlink Power Assignment

- If $\tilde{\mathbf{p}}$ is a global maximizer of the optimization problem, then

$$\begin{aligned} \mathcal{S}^{DL}(\tilde{\mathbf{U}}, P_{max}) &= \frac{\mathsf{SINR}_k^{(DL)}(\tilde{\mathbf{U}}, \tilde{\mathbf{p}})}{\gamma_k} \\ P_{max} &= \|\tilde{\mathbf{p}}\|_1 \end{aligned} \tag{18}$$

- Further we can elaborate Eq (18) as

$$\tilde{\mathbf{p}} \frac{1}{\mathcal{S}^{DL}(\tilde{\mathbf{U}}, P_{max})} = \mathbf{DG}(\tilde{\mathbf{U}})\tilde{\mathbf{p}} + \mathbf{Dn} \tag{19a}$$

$$\frac{1}{\mathcal{S}^{DL}(\tilde{\mathbf{U}}, P_{max})} = \frac{1}{P_{max}} \mathbf{1}^{\mathsf{T}} \mathbf{DG}(\tilde{\mathbf{U}})\tilde{\mathbf{p}} + \frac{1}{P_{max}} \mathbf{1}^{\mathsf{T}} \mathbf{Dn} \tag{19b}$$

- We can form eigen-system as

$$\underbrace{\begin{bmatrix} \mathbf{DG}(\tilde{\mathbf{U}}) & \mathbf{Dn} \\ \frac{1}{P_{max}} \mathbf{1}^{\mathsf{T}} \mathbf{DG}(\tilde{\mathbf{U}}) & \frac{1}{P_{max}} \mathbf{1}^{\mathsf{T}} \mathbf{Dn} \end{bmatrix}}_{\Upsilon(\tilde{\mathbf{U}}, P_{max})} \begin{bmatrix} \tilde{\mathbf{p}} \\ 1 \end{bmatrix} = \frac{1}{\mathcal{S}^{DL}(\tilde{\mathbf{U}}, P_{max})} \begin{bmatrix} \tilde{\mathbf{p}} \\ 1 \end{bmatrix} \tag{20}$$

Duality-Downlink Power Assignment

- The solution for SINR balancing problem (17) is now given as

$$\mathcal{S}^{DL}(\tilde{\mathbf{U}}, P_{max}) = \frac{1}{\lambda_{\mathsf{max}}\big(\Upsilon(\tilde{\mathbf{U}}, P_{max})\big)} \qquad (21)$$

- Similarly for uplink, and $\tilde{\mathbf{q}}_{\mathsf{ext}} = \begin{pmatrix} \tilde{\mathbf{q}} \\ 1 \end{pmatrix}$

$$\underbrace{\begin{bmatrix} \mathbf{D}\mathbf{G}^{\mathsf{T}}(\tilde{\mathbf{U}}) & \mathbf{D}\mathbf{n} \\ \frac{1}{P_{max}}\mathbf{1}^{\mathsf{T}}\mathbf{D}\mathbf{G}^{\mathsf{T}}(\tilde{\mathbf{U}}) & \frac{1}{P_{max}}\mathbf{1}^{\mathsf{T}}\mathbf{D}\mathbf{n} \end{bmatrix}}_{\Lambda(\tilde{\mathbf{U}}, P_{max})} \tilde{\mathbf{q}}_{\mathsf{ext}} = \lambda_{\mathsf{max}}\big(\Lambda(\tilde{\mathbf{U}}, P_{max})\big)\tilde{\mathbf{q}}_{\mathsf{ext}} \qquad (22)$$

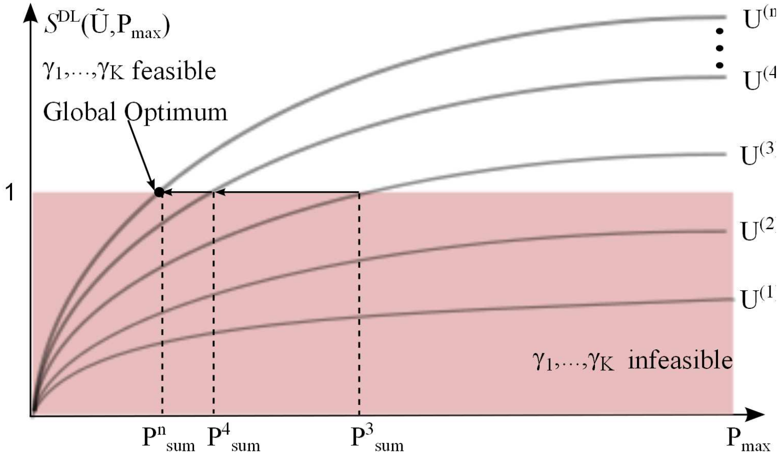
Duality-SINR Balancing

- For a given $\mathbf{q}_{\text{ext}}$, the cost function $\lambda(\mathbf{U}, \mathbf{q}_{\text{ext}})$ is minimized by independent maximization of the uplink SINRs.
- The optimal $\mathbf{u}_k$ could be now calculated as

$$\hat{\mathbf{u}}_k = \arg\max_{\mathbf{u}_k} \frac{\mathbf{u}_k^{\mathsf{H}} \overbrace{\mathbf{h}_k \mathbf{h}_k^{\mathsf{H}}}^{\mathbf{H}_k} \mathbf{u}_k}{\mathbf{u}_k^{H} \underbrace{\left(\sum_{j=1, j \neq k}^{K} q_j \hat{\mathbf{h}}_j \hat{\mathbf{h}}_j^{\mathsf{H}} + \sigma_{z_k}^2 \mathbf{I} \right)}_{\mathbf{W}_k} \mathbf{u}_k}. \quad (23)$$

- Eq. (23) is maximizing Rayleigh quotient problem.
- Eq. (23) is solved via dominant generalized eigen-vectors of matrix pairs $(\mathbf{H}_k, \mathbf{W}_k)$.

Power Minimization Algorithm



WHAT IF CHANNEL IS NOT PERFECT?

System Model

- MISO channel setting with N_t transmit antennas and K users with single antenna.
- Consider the transmit beamforming vector $\mathbf{u}_k$.
- Transmit signal from BS

$$\mathbf{x} = \sum_{k=1}^{K} \mathbf{u}_k s_k, \tag{24}$$

- Received signal at user k

$$y_k = \mathbf{h}_k^{\mathsf{H}} \left(\sum_{j=1}^{K} s_j \mathbf{u}_j \right) + z_k, \tag{25}$$

- The instantaneous SINR for user k

$$\mathsf{SINR}_k(\Phi_k) = \frac{|\mathbf{h}_k^{\mathsf{H}} \mathbf{u}_k|^2}{\sum_{j \neq k} |\mathbf{h}_k^{\mathsf{H}} \mathbf{u}_j|^2 + \sigma_{z_k}^2}. \tag{26}$$

Channel Error

- Common assumption:

Channel Error

- Common assumption:
  BS has perfect knowledge of channel

Channel Error

- Common assumption:
  BS has perfect knowledge of channel
- Reality:

Channel Error

- Common assumption:
  BS has perfect knowledge of channel
- Reality:
  No perfect CSIT available and constitute error within.

Channel Error

- Common assumption:
  BS has perfect knowledge of channel
- Reality:
  No perfect CSIT available and constitute error within.
- Causes:

Channel Error

- Common assumption:
  BS has perfect knowledge of channel
- Reality:
  No perfect CSIT available and constitute error within.
- Causes:
  Estimation errors and feedback delays.

Channel Error

- Common assumption:
  BS has perfect knowledge of channel
- Reality:
  No perfect CSIT available and constitute error within.
- Causes:
  Estimation errors and feedback delays.
- Effects:

Channel Error

- Common assumption:
  BS has perfect knowledge of channel
- Reality:
  No perfect CSIT available and constitute error within.
- Causes:
  Estimation errors and feedback delays.
- Effects:
  BSs cannot predict exactly the required SINR at the users.

Channel Error

- Common assumption:
  BS has perfect knowledge of channel
- Reality:
  No perfect CSIT available and constitute error within.
- Causes:
  Estimation errors and feedback delays.
- Effects:
  BSs cannot predict exactly the required SINR at the users.
- What can be done?

Channel Error

- Common assumption:
  BS has perfect knowledge of channel
- Reality:
  No perfect CSIT available and constitute error within.
- Causes:
  Estimation errors and feedback delays.
- Effects:
  BSs cannot predict exactly the required SINR at the users.
- What can be done?
  Estimate SINR under imperfect CSIT

Channel Error

- We model the imperfect CSIT as

$$\mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_{\mathbf{h}_k}, \tag{27}$$

$\hat{\mathbf{h}}_k$ is the estimated channel, $\mathbf{e}_{\mathbf{h}_k}$ is the respective channel error.

- The SINR with channel error incorporated will be

$$\Phi_{k_{(\mathsf{error})}} = \frac{|(\hat{\mathbf{h}}_k + \mathbf{e}_{\mathbf{h}_k})^{\mathsf{H}}\mathbf{u}_k|^2}{\sum_{j \neq k} |(\hat{\mathbf{h}}_k + \mathbf{e}_{\mathbf{h}_k})^{\mathsf{H}}\mathbf{u}_j|^2 + \sigma_{z_k}^2}. \tag{28}$$

Channel Error

- We model the imperfect CSIT as

$$\mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_{\mathbf{h}_k}, \tag{27}$$

$\hat{\mathbf{h}}_k$ is the estimated channel, $\mathbf{e}_{\mathbf{h}_k}$ is the respective channel error.

- The SINR with channel error incorporated will be

$$\Phi_{k_{(\text{error})}} = \frac{|(\hat{\mathbf{h}}_k + \mathbf{e}_{\mathbf{h}_k})^{\mathsf{H}}\mathbf{u}_k|^2}{\sum_{j \neq k} |(\hat{\mathbf{h}}_k + \mathbf{e}_{\mathbf{h}_k})^{\mathsf{H}}\mathbf{u}_j|^2 + \sigma_{z_k}^2}. \tag{28}$$

- The problem again? We still do not know the errors.

SINR Estimate under imperfect CSIT

- The estimated received signal

$$\hat{y}_k = \hat{\mathbf{h}}_k^{\mathsf{H}} \mathbf{u}_k s_k + \mathbf{e}_{h_k} \mathbf{u}_k s_k + \sum_{j=1, j \neq k}^{K} \hat{\mathbf{h}}_k^{\mathsf{H}} \mathbf{u}_j s_j + \sum_{j=1, j \neq k}^{K} \mathbf{e}_{h_k}^{\mathsf{H}} s_j \mathbf{u}_j + z_k,$$

$$= \underbrace{\hat{\mathbf{h}}_k^{\mathsf{H}} \mathbf{u}_k s_k}_{\text{estimated transmit signal}} + \overbrace{\sum_{j=1, j \neq k}^{K} \hat{\mathbf{h}}_k^{\mathsf{H}} \mathbf{u}_j s_j}^{\text{estimated interference}} + \underbrace{\sum_{k=1,}^{K} \mathbf{e}_{h_k}^{\mathsf{H}} s_k \mathbf{u}_k}_{\text{unknown interference}} + z_k.$$

SINR Estimate under imperfect CSIT

- The estimated received signal

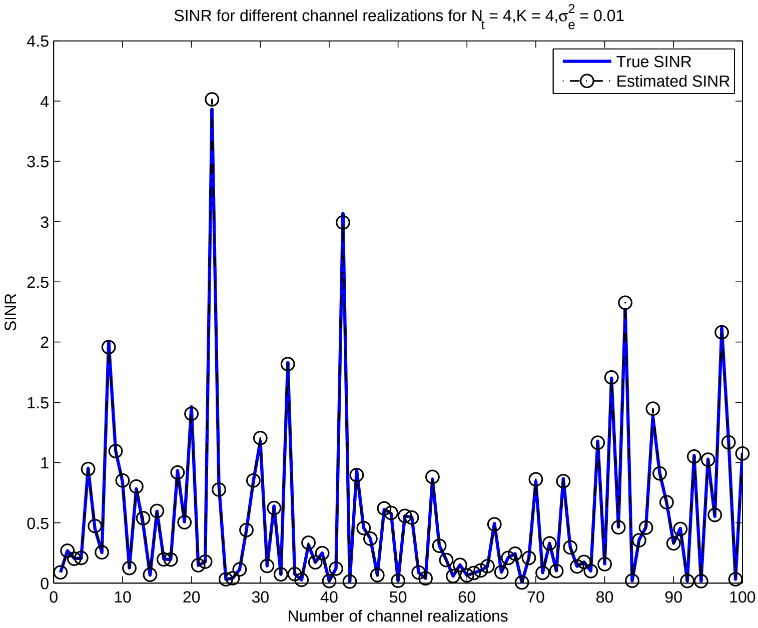
$$\hat{y}_k = \hat{\mathbf{h}}_k^{\mathsf{H}}\mathbf{u}_k s_k + \mathbf{e}_{h_k}\mathbf{u}_k s_k + \sum_{j=1, j\neq k}^{K} \hat{\mathbf{h}}_k^{\mathsf{H}}\mathbf{u}_j s_j + \sum_{j=1, j\neq k}^{K} \mathbf{e}_{h_k}^{\mathsf{H}} s_j \mathbf{u}_j + z_k,$$

$$= \underbrace{\hat{\mathbf{h}}_k^{\mathsf{H}}\mathbf{u}_k s_k}_{\text{estimated transmit signal}} + \overbrace{\sum_{j=1, j\neq k}^{K} \hat{\mathbf{h}}_k^{\mathsf{H}}\mathbf{u}_j s_j}^{\text{estimated interference}} + \underbrace{\sum_{k=1,}^{K} \mathbf{e}_{h_k}^{\mathsf{H}} s_k \mathbf{u}_k}_{\text{unknown interference}} + z_k.$$

- The biased estimated of SINR will be now

$$\hat{\Phi}_{k_{(\text{biased})}} = \frac{|\hat{\mathbf{h}}_k^{\mathsf{H}}\mathbf{u}_k|^2}{\sum_{j=1, j\neq k}^{K} |\hat{\mathbf{h}}_k^{\mathsf{H}}\mathbf{u}_j|^2 + \sigma_{\mathbf{e}_{\mathbf{h}_k}}^2 \mathsf{Trace}(\mathbf{U}\mathbf{U}^{\mathsf{H}}) + \sigma_{z_k}^2} \tag{30}$$

Robustness of Estimate



Comparison against unbiased

- The unbiased estimation of SINR

$$\hat{\Phi}_{k_{(\text{unbiased})}} = \frac{|(\hat{\mathbf{h}}_k)^{\mathsf{H}}\mathbf{u}_k|^2}{\sum_{j \neq k} |(\hat{\mathbf{h}}_k)^{\mathsf{H}}\mathbf{u}_j|^2 + \sigma_{z_k}^2}. \tag{31}$$

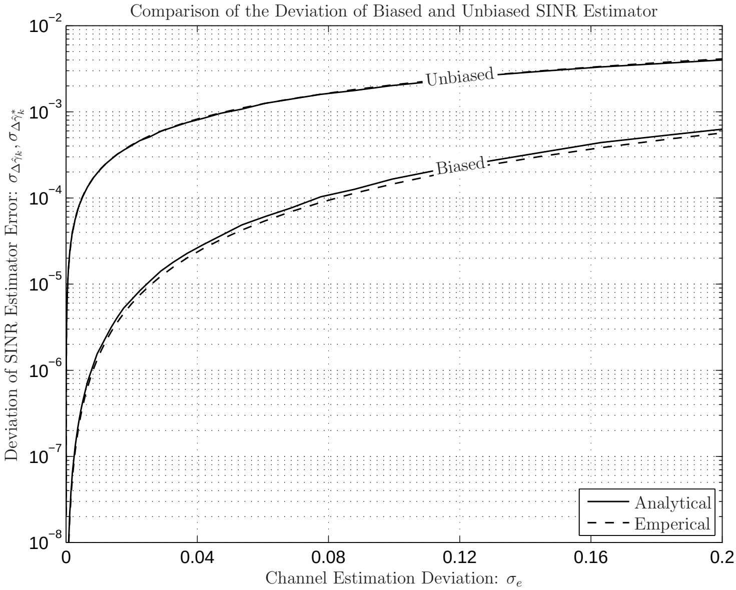
- Consider estimation errors of unbiased and biased estimations.

$$\Delta\hat{\Phi}_{k_{(\text{unbiased})}} \triangleq \hat{\Phi}_{k_{(\text{unbiased})}} - \Phi_k,$$
$$\Delta\hat{\Phi}_{k_{(\text{biased})}} \triangleq \hat{\Phi}_{k_{(\text{biased})}} - \Phi_k,$$

- The deviation of error holds following equality

$$\sigma_{\Delta\hat{\Phi}_{k_{(\text{biased})}}} = \frac{\sigma_{\mathbf{e}_{\mathbf{h}_k}}}{\sqrt{2}\left((1+\sigma_{\mathbf{e}_{\mathbf{h}_k}}^2) + \frac{(\sigma_{z_k}^2 - \text{Trace}(\mathbf{u}_k\mathbf{u}_k^{\mathsf{H}}))}{\text{Trace}(\mathbf{U}\mathbf{U}^{\mathsf{H}})}\right)}\sigma_{\Delta\hat{\Phi}_{k_{(\text{unbiased})}}}. \tag{33}$$

Comparison against unbiased



Back to Power Minimization Problem

- The SINR constraint in presence of channel error is

$$p_k G_{kk} \geq \gamma_k \left(\sum_{j=1, j\neq k}^{K} p_j G_{kj} + \sum_k p_k \sigma^2_{\mathbf{e}_{\mathbf{h}_k}} + \sigma^2_{z_k} \right). \qquad (34)$$

where $G_{kj} = |\hat{\mathbf{h}}_k^H \mathbf{u}_j|^2$.

- We now define for $a_k = \sigma^2_{\mathbf{e}_{\mathbf{h}_k}}$

$$\mathbf{D} = \begin{bmatrix} \frac{\gamma_1}{G_{11}} & 0 & \cdots & 0 \\ 0 & \frac{\gamma_2}{G_{22}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\gamma_K}{G_{KK}} \end{bmatrix}, \; \mathbf{G_I} = \begin{bmatrix} a_1 & (G_{12}+a_2) & \cdots & G_{1K}+a_K \\ (G_{21}+a1) & a_2 & \cdots & G_{2K}+a_K \\ \vdots & \vdots & \ddots & \vdots \\ (G_{K1}+a_1) & (G_{K2}+a_2) & \cdots & a_K \end{bmatrix}. \qquad (35)$$

- In matrix form, the SINR constraint can be written as

$$(\mathbf{I} - \mathbf{D}\mathbf{G_I})\mathbf{p} \geq \mathbf{D}\mathbf{n}, \qquad (36)$$

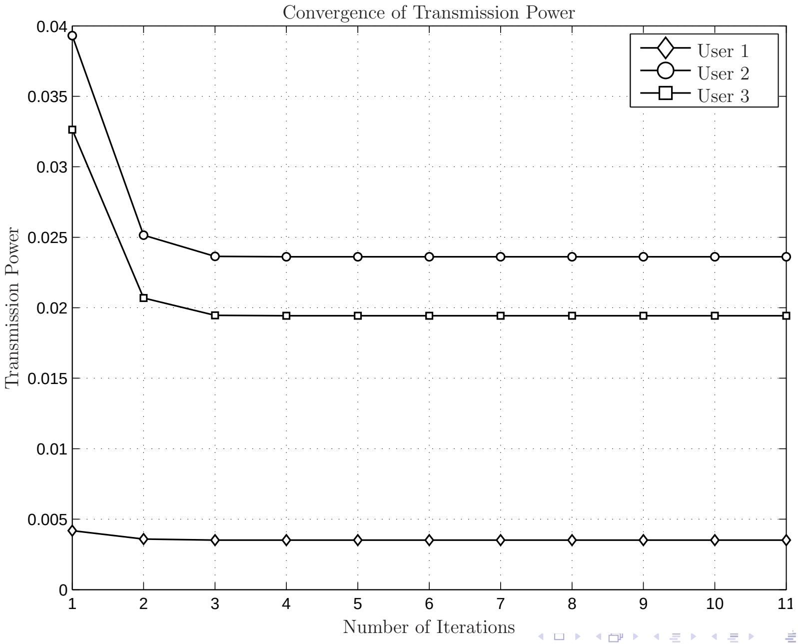
Power Minimization Problem

- The optimal $\mathbf{u}_k$ can be calculated as 'maximizing Rayleigh quotient' problem

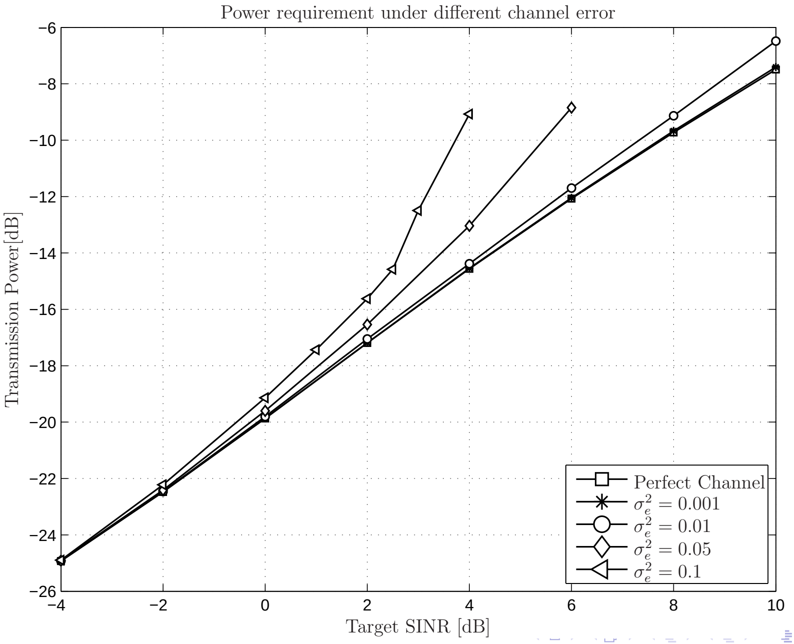
$$\hat{\mathbf{u}}_k = \arg\max_{\mathbf{u}_k} \frac{\mathbf{u}_k^H \underbrace{\hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^{\mathsf{H}}}_{\mathbf{H}_k} \mathbf{u}_k}{\mathbf{u}_k^{\mathsf{H}} \underbrace{\left(\sum_{j=1, j\neq k}^{K} q_j \hat{\mathbf{h}}_j \hat{\mathbf{h}}_j^{\mathsf{H}} + \sigma_{z_k}^2 \mathbf{I} + \sum_k q_k \sigma_{\mathbf{e}_{\mathbf{h}_k}}^2 \mathbf{I} \right)}_{\mathbf{W}_k} \mathbf{u}_k}. \quad (37)$$

- The optimal $\mathbf{u}$'s are given by the dominant generalized eigenvectors of matrix pairs $(\mathbf{H}_k, \mathbf{W}_k)$.

Results: Convergence of Algorithm



Results: Comparison of different errors



Future Works

- The stochastic SINR (for high SNR) for channel error is given as

$$\hat{\Phi}_{k_{(\text{biased})}} = \frac{|\hat{\mathbf{h}}_k^{\mathsf{H}} \mathbf{u}_k|^2}{\sum_{j=1, j\neq k}^{K} |\hat{\mathbf{h}}_k^{\mathsf{H}} \mathbf{u}_j|^2 + \underbrace{\mathbf{e}_{h_k}^{\mathsf{H}} \mathbf{U}\mathbf{U}^{\mathsf{H}} \mathbf{e}_{h_k}}_{X_1} + \sigma_{z_k}^2} \tag{38}$$

- X_1 is gamma distributed $\Gamma(x_1; a, b)$ with shape parameter $a \geq 0$, and scale parameter $b \geq 0$.
- The density function is given as

$$p_{X_1}(x_1; a, b) = \frac{1}{b^a \Gamma(a)} x_1^{a-1} e^{\left(\frac{-x_1}{b}\right)}. \tag{39}$$

- The parameters a and b is given as

$$a = \frac{(\text{Tr}[\mathbf{U}\mathbf{U}^{\mathsf{H}}])^2}{\text{Tr}[(\mathbf{U}\mathbf{U}^{\mathsf{H}})^2]} \tag{40a}$$

$$b = \frac{\sigma_{e_{\mathbf{h}_k}}^2 \text{Tr}[(\mathbf{U}\mathbf{U}^{\mathsf{H}})^2]}{\text{Tr}[\mathbf{U}\mathbf{U}^{\mathsf{H}}]} \tag{40b}$$

Future Works-Estimating Distribution

- Based on distribution of X_1, the estimated SINR is distributed as

$$p_{\hat{\Phi}_k}(\hat{\phi}, a, b) = \frac{c}{\hat{\phi}^2} \frac{\left(\frac{c}{\hat{\phi}} - \delta\right)^{a-1} \exp\left(\frac{\frac{c}{\hat{\phi}} - \delta}{b}\right)}{b^a \Gamma(a)}, \tag{41}$$

where the constants c and δ are given as

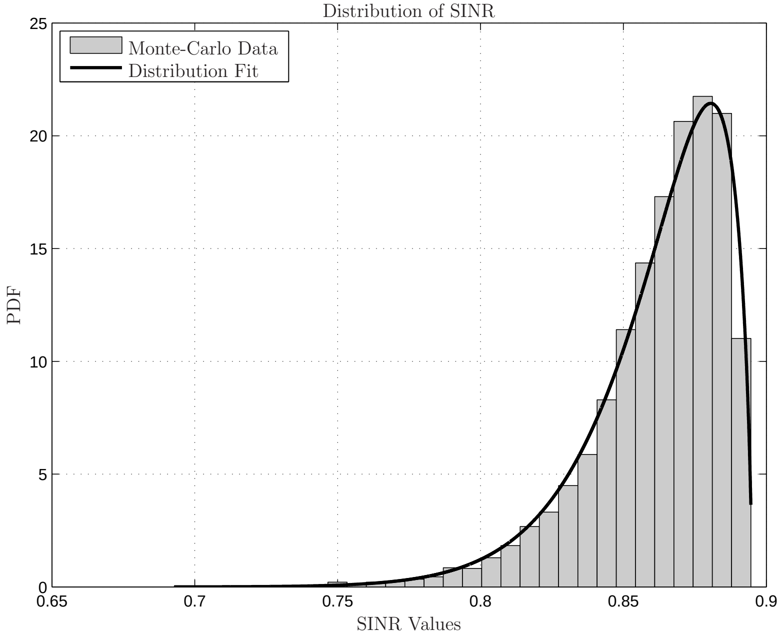
$$c = |\hat{\mathbf{h}}_k^{\mathsf{H}} \mathbf{u}_k|^2, \tag{42a}$$

$$\delta = \sum_{j=1, j\neq k}^{K} |\hat{\mathbf{h}}_k^{\mathsf{H}} \mathbf{u}_j|^2 + \sigma_{z_k}^2, \tag{42b}$$

- As an example, the probabilistic constrained power minimization problem can be written as

$$\mathcal{S}_{\mathsf{Prob(MISO)}} = \begin{cases} \underset{\mathbf{p}}{\text{minimize}} & \sum_{k=1}^{K} \|\mathbf{u}_k\|^2 \\ \text{subject to} & \text{Prob}\{\text{SINR}_k \geq \gamma_k\} \geq 1 - \rho_k \quad 1 \leq k \leq K, \end{cases} \tag{43}$$

Estimating Distribution of SINR



Future Works

- The channel error case could be now extended to probabilistic approach.
- Consider different channel errors: uncertainty region bounded / not bounded.
- Extend the 'Power Minimization', 'Max-min SINR', 'Sum-Rate Maximization' problem to MIMO multi-cell cases.
- Implement 'Centralized' and 'Decentralized' processing schemes.
- Compare all three problems under same umbrella.

Thank You

Questions?

Suggestions!